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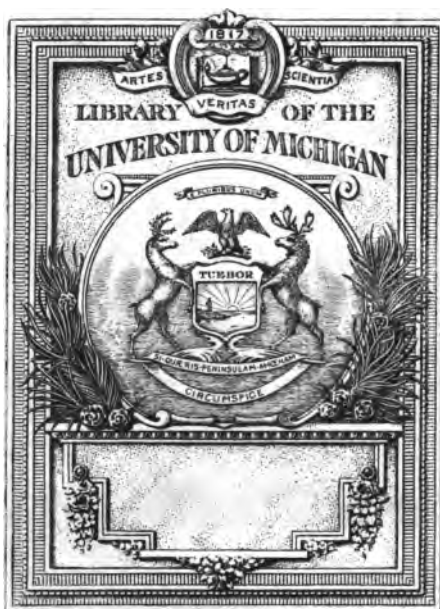
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A
TREATISE
OF
PERSPECTIVE
DEMONSTRATIVE
AND
PRACTICAL.

Illustrated with Copper CUTTS.

By HUMPHRY DITTON,
Master of the *New Mathematical*
School, in Christ's Hospital.

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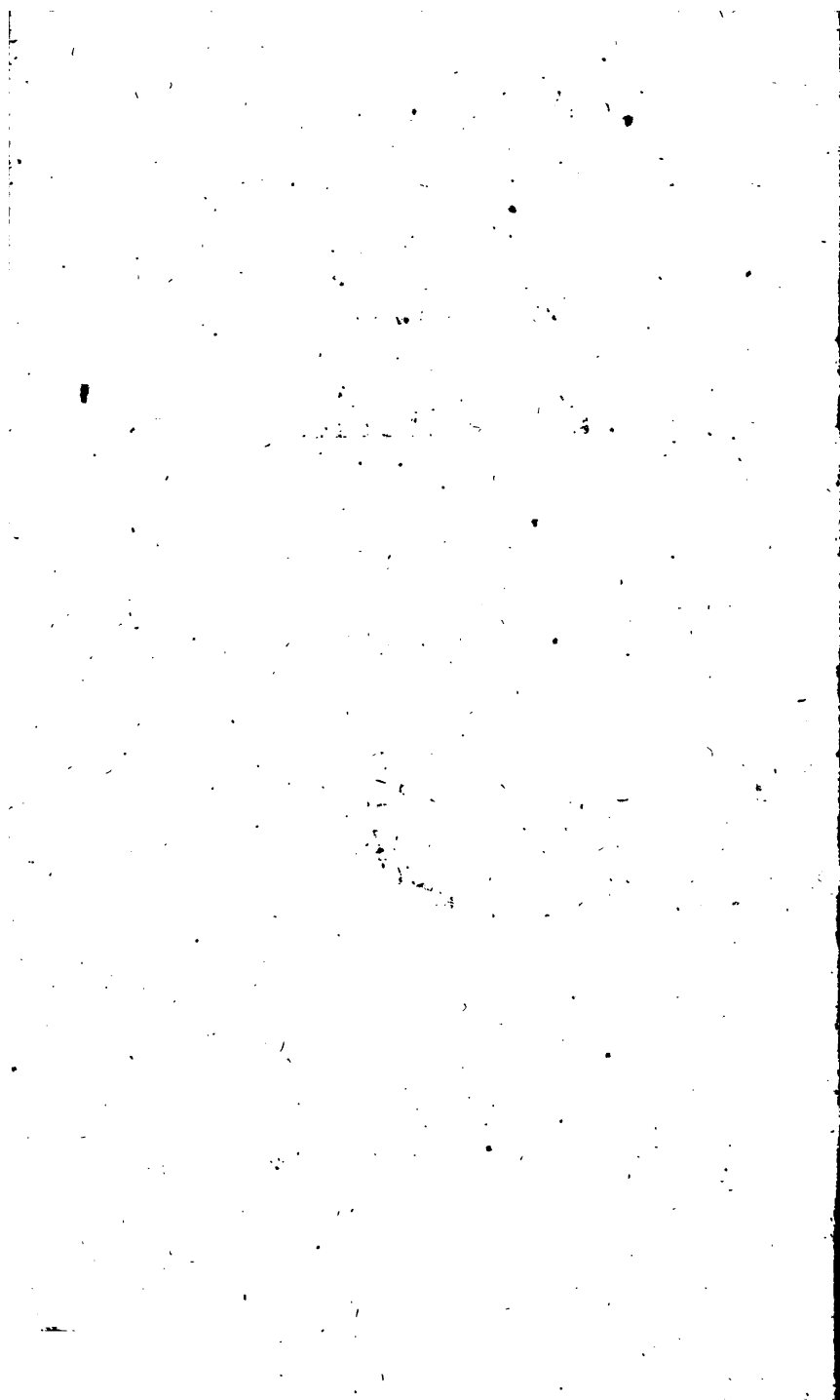
To the Honourable
Francis Nicholson, Esq;
General of Her MAJESTY's
FORCES
IN
North America;

A
True Patriot, a Gentleman, and a Friend,
To whom his COUNTRY is indebted
for many signal Services Abroad, as
LEARNING is, for a Generous En-
couragement at Home;

THIS
Treatise of PERSPECTIVE
(Design'd for the Use of the *NEW MA-*
THEMATICAL-SCHOOL in
Christ's-Hospital) is humbly Dedicated,

In Testimony of
That profound RESPECT
Which is (and ever will be) paid to him

BY THE
AUTHOR.



P R E F A C E

T O T H E
R E A D E R.

SINCE Custom has made it a Point of Civility to the Reader, as well as a sort of Ornament to a Book, to introduce it with some Prefatory Discourse; I shall endeavour to make the following one as *Useful* as I can, by discoursing of something by which I may inform the Reader, at the same time that I pay him the usual Complement.

And therefore as the Design of the Ensuing Treatise, is to explain the Nature and Properties, of one particular Sort or Kind of PROJECTION; so I propose here to explain, the Nature of PROJECTION IN GENERAL, with its several Kinds, and their Uses and Differences one from another; and this, as far as the Bounds that are here set me will permit.

PROJECTION, is the Transcription or Delineation, of an Object upon a Plane. Or rather; 'Tis the Figure, mark'd or trac'd out upon a Plane, by a moveable Line, extended from the EYE, as a common Poles or Centre, to the several Points of an Object. Upon this Account, 'tis called by some, by the Name of SECTION, and that not improperly; for that Figure,
b Image

Image, or Representation of an Object upon a Plane, which we call **PROJECTION**; is no other than the **SECTION** of the Visual *Cone*, *Pyramid*, *Cylinder*, or *Prism*, by the Plane on which the said Figure is design'd. It's easily understood from hence, how great a Variety of Projection arises, from the various Positions, both of the Eye, the visible Object, and the Plane it self. That as it must needs be vastly different, if the Eye and Plane continuing their Situation, the Object changes from a direct Position to an oblique; so likewise it must be, if the Eye and Object remaining as they were, the Plane be mov'd from one Situation to another. And then if the Plane, be between the Object and the Eye; the Projected Figure will be less, the nearer the Plane is to the Eye, and the greater, the further off; supposing the Eye and Object to be fix'd, and the Plane to move: As also that the projected Figure will be greater, the further the Eye is from the Plane, and less, the nearer it is to it; supposing the Object and Plane to be fix'd, and the Eye to move backwards or forwards. On the other hand; that if the Object, be between the Plane and the Eye: Then the further the Object is from the Plane, the bigger its Projection is, and the nearer, the less; supposing the Plane and the Eye to retain their Positions, and the Object to move; or that the farther the Eye is from the Plane, the less the Projection of the Object is, and the nearer, the bigger; supposing the Object and Plane to retain their Positions, and the Eye to move.

These things are obvious, upon the drawing of three or four strait Lines. And therefore passing this, I think it not improper to observe in the next Place, that we ought to conceive a Difference, between

between the PROJECTION, and the bare APPEARANCE, of an Object to the Eye. For the Situation of the *Object*, and of the *Eye*, continuing, the APPEARANCE is still the same: But tho' the *Eye* and *Object* should retain their Positions, yet if the *Plane* alters its Situation, the PROJECTION will not be the same, but very different. So that these Two are not entirely the same; nor are the Words therefore to be used promiscuously, as Terms perfectly equivalent, and that signify one and the same thing. APPEARANCE depends only upon the Relation of two things to each other, viz. the *Object*, and *Eye*: But PROJECTION besides those Two, takes in the Consideration of a *Plane*, which besides a vast Variety in it self, introduces a considerable Difference between it and the other. Yet after all; Projection is no more than *Relative Appearance*; that is, such as results, from this or that particular Situation, of *Eye*, *Object*, and *Plane*, altogether. And 'tis this particular Consideration of a *Plane* likewise, that distinguishes this Science, from what we commonly call, SIMPLE or DIRECT OPTICS. For as there, we consider Quantities purely as VISIBLE, or as the Objects of Vision; so here, they are consider'd, as Visible, with Respect to, a certain Plane, lying in this or that particular Position.

The different Kinds or Species of Projection, must be taken, from the various placing, either of the *Object*, the *Plane*, or the *Eye*.

That they ought not to be deriv'd from the Positions of the *Object*; is plain. Because, tho' very different Projections will arise from hence, yet they would be Infinite; even as many, as there are Positions, that the Object may be plac'd in,

Besides; from a different POSITION, arises a different APPEARANCE to the Eye: And since Projection, is only transcribing the Object as it *appears*; we should thus, rather be projecting so many *several different Objects*, than making the several different Projections of one and the same Object.

Neither ought this Distinction, to be taken from the various *Positions of the Plane*, which in general can be but Three, viz, *Perpendicular, Parallel, or Oblique*, to a Ray, let fall perpendicularly, from the Eye, to the Object.

'Tis true, that there arise from hence, three very different Projections; and a Man may, if he pleases, call them different *Kinds of Projections* too: But however, it would be of little Use or Advantage to distinguish them thus; nay it would be (without farther Limitations and Conditions) an obscure, doubtful and ambiguous Way of giving an Account of a Projection, to say, *It was such a one, that the primary visual Ray, was at Right Angles, or not at Right Angles, to the Plane, on which the Projection was made.*

A Man by this, might possibly, in many Cases, understand one Sort of Figure, when in Reality, it was quite another that was intended.]

Certainly the Distinction of the *Kinds of Projection*, ought to be taken from that Principle, and that only, which will infer the most compleat, comprehensive, and easily conceivable Difference, between the Members so distinguish'd; and that Principle must of necessity be, *the various Distance of the Eye.*

This takes in and accounts for all; and introduces a clear and distinct Notion, of Three Kinds of Projection, vastly different from one another.

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Nor can there, upon this Principle, possibly be any more than *Three*; since there can be but a threefold Variety in that Article of Distance. For the Eye may be suppos'd, either to be *Infinitely remov'd*, or *Infinitely near*, (or in *Contact*, as they express it) or else at some *just and moderate Distance*.

Accordingly we have (what the Writers of this Science, have call'd by the Name, of the) ORTHOGRAPHICAL, STEREOGRAPHICAL, and SCENOGRAPHICAL, Projections: Of each of which, we shall speak something in their Order.

In the *Orthographick* Projection, we commonly say, the Eye is suppos'd to be at an *Infinite Distance*; which is not to be understood *strictly*, but *comparatively* so; or in a more rude and vulgar way of Speech, for an *Immoderate or very great Distance*, and which with Respect to our *Ordinary Views* (which are taken at *small Distances*) may well enough be call'd Infinite. We may fairly reckon that to be an *Immoderate* (and in this Sense therefore an *Infinite*) Distance, when the *Parts* of an Object, which in *reality*, bear a very considerable Proportion to the *whole*; do notwithstanding disappear and lie hid, so that we can't discern *Excesses and Defects*, or make Comparisons between them, as we could easily do, at some other Stations less distant from that Object.

And therefore, this *Infinite Distance* we speak of, is so far from consisting in *Indivisibili*, or being only Immense Distance; that it admits of a great Latitude; nay is capable of *Infinite Variety*, according to the *Magnitude and Extent* of the Objects view'd or consider'd. That same Distance, which with Respect to a *very small Object*, may be *Immoderate and Excessive*; with Respect to a *Great one*, may be *just and Moderate enough*. Or a *very small* Distance,

Distance, in Comparifon to a minute Object, may be immoderate; when a vastly great one may be the Contrary, with Refpect to an Object of proportionally large Dimensions. The *Moon's* Distance from the *Earth* is properly enough ftill'd *Infinite*, in Comparifon to fome petty Meafures of Length and Distance, in common Ufe here amongst us. But yet it is not fo, with Refpect to the *Semidiameter*, of the *Terreftrial Globe*. For we find (for Example) that the Appearances of *Solar Eclipses* are very different, at the very fame Moment of *Absolute Time*; to People that obferve them, from different Parts of the *Globe*: Which fhews, that the *Semidiameter* of our *Earth* is far enough from being as a *Point*, with Refpect to the Distance of the *Eclipsing Luminary*, and does indeed bear fome confiderable Proportion thereto; and this Proportion is commonly exprefs'd in round Numbers, by that of 1 to 60. However this fame *Semidiameter* of the *Terreftrial Globe*, bears no *fenfible* Proportion, to the *Sun's* Distance from it; which therefore is in our Senfe, an *Infinite* Distance. Hence we take the *Sun's* Rayes as *Parallel*, and determine the *Foci* of them in *Refracting or Reflecting Glaffes*, as for Rayes that are really *Parallel*; and that without confiderable Error. We fuppofe the *Sun* to enlighten *Half* the *Globe* of our *Earth*; when as in *Geometrical* Strictnefs, 'tis certain that he enlightens *more* than a *Hemifphere*. But then, as one and the fame *Luminary*, enlightning one and the fame *Spherical Body* which is *lefs* than that *Luminary*; enlightens a *lefs* Portion of it (tho' always *more* than a *Hemifphere*) at a *greater* Distance, than it does at a *lefs* Distance; fo upon the Account of an *Immoderately* great Distance, between the two Bodies, the enlightened Part will approach

so near to a Hemisphere, or rather, the Excess of the Enlightned Part above a Hemisphere, will be so far diminish'd; that no sensible Difference will arise.

And this is the Case, with Respect to the Sun, and the Globe we live on; upon which Score (tho' it be not *Mathematically True*) we say, that *Half the Latter* is enlightned by the *Former*. So also, we take the *Shadows of Equidistant Gnomons*, to be *Parallel to one another*; and say, that 'tis the same thing, whether *Dials* are plac'd on the Surface, or at the *Centre of the Earth*; whereas rigorously Speaking, neither are, nor can, the *Shadows of such Gnomons* be parallel (unless in one Case, when the *Gnomons themselves*, are dispos'd parallel to the *Plane*, on which the *Shadows* are receiv'd) nor are *Dials* exact, plac'd any where but at the *Centre*; where, and where only, the *Stile* truly answers to the *Axis of the Globe*, and the *Planes* themselves, to the *Planes of the Great Circles*, which they represent.

But to proceed. It is upon the Account of this suppos'd Infinite Distance of the Eye; that all ORTHOGRAPHICK Projections are design'd by *Parallel Rayes*. Indeed in Nature, there is not, nor can be any such thing, as *Parallel Radiation*; either from a REAL, or FICTITIOUS Radiant, such as is an Eye; but the *Angles* becoming Indefinitely Small, and therefore Insensible, when the *Distance* is Indefinitely Great; we therefore take the *Projecting Rayes* in this Case, as *Parallel*, and proceed accordingly.

From hence it is, that in Projections of the *Sphere* this way, all *Circles* both *Great* and *Small*, the *Planes* of which, are not at Right Angles, to the
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Plane

Plane of that Circle, on which the Projection is made ; do all fall into the Form of *Ellipses*.

All *little* Circles, as also all *great* ones, which are perpendicular to the said Plane ; (such as are the *Equator*, *Ecliptick* and *Horizon* in the Common *Analemma*) are represented by *straight Lines*. Nor can there be any sort of Line, *Circular* here ; except only the Periphery of that Circle, in a Line drawn thro' the Pole of which, the Eye is suppos'd to be plac'd, at an Infinite Distance ; Or in other Words ; the Circle we project upon : as (for Example) the *Solstitial Colure*, in the *Instrument* just now mention'd.

From hence likewise it is, that all *Arches*, being projected, into their *Right Sines* ; the Line of *Sines* is of so necessary Use, in describing and solving Problems, by this Sort of Projection. There are many Useful, and Noble Projections of the Sphere, made this way ; and particularly very curious, (I won't say the most practicable) Constructions of *Dials* to be drawn from thence. Yet it must be confess'd, that as the nice Description of *Ellipses*, is a troublesome and laborious Practice ; so there is an Inconvenience in that Respect, attending an *Orthographical* Projection, where a Problem requires an *Ellipse* to be describ'd ; which some of the very *Fine* ones do ; tho' most of the useful *Vulgar* ones do not, but may be done by Right Lines and Circles only.

Nor should we omit taking Notice, of that particular Inconvenience likewise, in *Orthographical* Projections, *viz.* The *extream* narrowing and crowding together of the Parts, toward the outside ; which is the unavoidable Consequence of the *Parallelism* of the *Rayes*. As common *Geometry* will convince any one, that divides the Circumference of a Circle in-

to a good Number of equal Parts, and draws Chords, thro' the opposite Correspondent Points.

This Sort of Projection, by *Parallel Rayes*, is very useful in other Cases, besides that of designing the Circles of the *Sphere* upon a Plane, for *Astronomical Purposes*.

In *Military Architecture*; the Draughts of *Fortifications*, are made this way, not only with more Ease and Expedition, but with most Convenience and Advantage too.

In *Civil Architecture*, *Orthography* properly signifies the upright Delineation of the *Front*: Thus *Vitruvius* defines it, *Orthographia est Erecta Frontis Imago*, *Lib. 1*. And by *Front* I presume is commonly intended, all that can be seen *directly*, at one single View; whether *inward* or *outward*, whether consisting of one Plane only, or of more. But this is a more restrain'd Sense and Application of the Word; for it denotes in general, a Delineation or Designation by *perpendicular Lines*; which comes up to the true Purport of the Term *ὀρθογραφία*. And it is after this way, that the *Plans* and *Elevations* of Buildings, are ordinarily drawn.

The *Ichnography* (or Plan) *ex. gr.* is an *Orthographick Projection*, on the *Ground Plane*; or, which is the same thing; 'tis the Section by a Plane parallel to the Horizon.

The *Profile*, is the same Sort of Projection upon a *Vertical Plane*, parallel to that, by which the Body is suppos'd to be cut through. Sometimes the *entire Section* it self (in which not only the bare out-Lines, but also the Thickness of the *Walls* appears) is represented this Way.

All these Projections are design'd, by *Perpendiculars* let fall, from the several Points of the Object,

to the Plane or Table, on which the Figure is to be drawn. For which Reason they must all of them, necessarily be *Similar* to their respective *Primitive Figures*; being made (as they are suppos'd to be) on Planes *parallel* to those, in which the *Originals*, or *Primitive Figures* are conceiv'd to lie. Farther, tho' *Height* and *Thicknes*s, may well be represented this Way; yet there can be no Expression of *Depth* or *Profundity*. The Nature of the Projection, will not allow any Representation of *this Dimension*. However (as *Vitruvius* intimates, *Lib. 1. Ch. 2.*) it may be allowable to remedy this, by *Shading* or *Colouring* what is thus describ'd *Orthographically* upon a Plane; by which Means the Elevation and Depression, and so the due Distinction of Parts, may be exhibited; tho' it can never possibly be done, by the *bare Lineaments*, or Geometrick Design.

But to go on with our Discourse.

The STEREOGRAPHICK Projection, comes next to be consider'd. This is that, which is said to be, *Ex Oculi contactu*, because the Eye in this Sort of Projection, is conceiv'd to be posited, on the very Surface of the Body or Figure to be projected. And there is this particular Advantage arising from thence, *viz. That, in the SPHERE, (about which this Projection is principally conversant) all the Parts are separately and distinctly represented; and that there is no one Point (excepting that only where the Eye is plac'd) whose Projection coincides, with the Projection of another Point.* For the Rayes drawn from the Eye, to the Points of the Spherick Surface, will cut the Plane on which the Projection is made, each in its own proper distinct Point. Indeed, in the Case of Bodies, that are contained under *Rectilineal*-figur'd Surfaces; where the Projections of several Points will

will be coincident with one another, and that because of the Rectilineal Surfaces; as if *ex. gr.* the Eye were plac'd in one of the *solid* Angles, of either of the *Regular* Bodies; the Projections, of the several Points of those Surfaces, whose Angles compose the Solid Angle where the Eye is fix'd, will be coincident with one another, because the said Points lie all in *strait* Lines. But in the Sphere, or other Solid, contain'd under a *Curve* Surface; it will be otherwise. It is from hence, that this Projection has its Name of STEREOGRAPHICK; because not only the *Ambit* or *Outside* of a Body is this way describ'd, but the *τὸ σῆμα*, the *Solidity*, or entire Content of it: As the Geometry of Solids, is for the same Reason called *σῆμα*.

To give the compleat and entire Figure, of a Body thus on a plane Superficies, is the peculiar Property of the *Stereographick Projection*; for neither the *Orthographick* nor the *Scenographick*, can possibly do this.

Besides; the Parts of the Projecture, in going from the Centre to the Circumference, which in the *Orthographick* Projection, are so crowd'd together; that they are the least fit for Use, where many times they ought to be of the most Use; these here, are gradually augmented, and that with no very excessive Increase, till we come to a Hemisphere; after which they are indeed, more immoderately augmented.

But then (which is likewise not only a noble Property, but a most considerable Ease and Advantage in this Projection, is, that) all the Circles of the Sphere, both *Small* and *Great* (except those *Great* ones only which pass thro' the Eye, and which are desigu'd by Right Lines) are represented here by *Circles*; and that as none of them else can be *strait*
Lines,

Lines, so neither can they be *Ellipses*; as they will be (some of them, both *Great* and *Little* ones) in the *Orthographick* Projection. Farther, all *Arches* being projected here, into their *Semitangents*; that *Line* becomes of as standing Use in this, as that of *Right Sines* is in the *Orthographick* Projection.

The *Inverse* of the *STEREOGRAPHICK* Projection; is that which is commonly call'd the *GNO-MONICAL* (as being that on which the ordinary Description of *Dials* is founded.)

I call it the *Inverse*, of the *STEREOGRAPHICK*, because of the *Reciprocal* Positions of the *Eye* and the *Plane*, in these two Projections.

For as *there*, an *Eye* plac'd somewhere on the Circumference of the Sphere, projects upon a *Plane* passing through the Centre; so *here*, an *Eye* plac'd in the Centre, projects upon a *Plane*, touching the Surface of the Sphere. Upon this Score 'tis, that *Arches* are here projected not into their *Semitangents*, as in the other, but into their *Tangents*,

All *Great* Circles fall into *strait* Lines.

All *Little* ones, parallel to the *Plane* of the Projection, come into *Circles*; and the rest, according to their various Positions, into the other *Conical* Sections.

This Projection, being not so vulgarly talk'd of, as the rest; I thought it would not be amiss, to give a little Explication of it here, in a *Figure* drawn for that Purpose. (See *Fig. 37.*)

Conceive the Sphere, whose Centre is O, and which is touched by an *Infinite Plane* in A, to be cut thro' its Centre, and the said Point of Contact, by another *Infinite Plane*; by which Means, the *Great Circle* which appears here, will be produc'd by the Section of the Sphere; the *Infinite Line* DAG for the

the Common Section of the two Infinite Planes; and the other Right Lines drawn in the Figure, will be the common Sections of the Planes of the Circles of the Sphere, both Great and Small, by the aforesaid cutting Plane.

It's plain that the *Great Circles* SX, QW, VR, &c. are projected into *Right Lines*; as all passing thro' the Eye at O. If the *Little* one KI be parallel to DG, then BE, is the projected Diameter of a *Circle*. But CE, into which PI is projected, is of Necessity, the longest Axis of an *Ellipse*; and so of all other *little Circles*, drawn under the same Conditions.

For the Triangles COE, POI, can never possibly be *Similar*; the *latter* being ever an *Isosceles*. So that there can be no *subcontrary Section* here; and therefore no *little Circle*, can fall into a *Circle*, if it does not lie parallel to the Plane DAG. Such a prodigious Difference, does the bare shifting the Place of the Eye, in these two Projections make; that whereas in the STEREOGRAPHICK, we have nothing but *subcontrary Sections*, in the GNOMONICK, we have none at all.

The Circle LN will be an *Hyperbola* upon the Plane DG, which cuts the Side ON of the Cone LON, produced beyond the Vertex O. The Circle HM will be a *Parabola*; for I suppose TOM to be parallel to DAG. And so of the rest: A Man may at Liberty determine the *Positions* of his little Circles, and so see what Sections they will be, when *Gnomonically* projected. I could shew a Method, something peculiar, for describing these Curves; but that's not my Work here, and besides those Practices are common enough; nay, 'tis as common now a-days for People to do them, as 'tis for them, not to understand one Word of the Demonstrations.

monstrations. This same Figure, will serve to shew the Grounds of another pretty curious Speculation in these Matters; and that is, *What Conick Sections are described by the Shaddows of the Stiles of Dials, at any time of the Year, in any given Place.*

For suppose VR the *Axe* of the World, QW the *Equator*, LN some parallel of *Declination*; DAG, the *Horizon* of any Place.

The Angle DEV is the *Latitude*, suppose $= n$ Degrees; the *present Declination* WN $= p$ Degrees; therefore the Angle NOF or NOR $= 90 - p$ Degrees. Now if $n =$ or $<$ or $>$ $90 - p$; the Shadow of the *Gnomon* upon the Plane DF, will at that Time, describe a *Parabola*, *Hyperbola*, or *Ellipsis*; as is most obvious from the various cutting of the *Cone* LON, by the Dial-plane DAG. If n were $= 90$, the Section becomes a *Circle*; and the Place whose *Horizon* DG is, is the *Pole* it self. This may be express'd in particular Examples, for particular *Latitudes*, and any Dial-planes at Liberty.

In the last Place of all, the SCENOGRAPHICK Projection comes to be consider'd. This proceeds (as they say) *Ex Justo & Moderato Oculi Intervallo*; the one of the other two Sorts of Projection being *ex Contactu*, and the other *ex Infinita Oculi distantia*.

What this *Just and Moderate* Distance is, is not so easily determin'd, though many have given their Rules for the fixing of it. Indeed speaking *Universally*, it is not determinable, in the very Nature of things: That being a moderate Distance, with Respect to one Eye and one Object, which is not so, with Respect to another; so that there
can

can be no settling that Point, but with Regard to those Conditions.

This Projection is of no Use, with Respect to the Representation of the Circles of the Sphere: ('Tis true, a Circle may be a Circle here; but it must be by subcontrary Position; unless it stands parallel to the Table.) But 'tis of most admirable Use in designing all Sorts of Solids and Surfaces, *Buildings, Walks, Rivers, Animals*, and in a Word, whatever appears in Nature, within the Limits of a proper Distance. And this it does the most to the *Life*, of any Sort of Projection whatsoever. 'Tis this Science, which teaches those pretty *Frauds in Vision*, which give us so much Pleasure, and make us even fond of being imposed upon. 'Tis from hence that *Painting, Sculpture*, and all the fine Arts of *Imitation*, derive their Force and Beauty.

And 'tis the Explication of this, in its *demonstrative* Grounds and Principles, as well as in all the necessary Branches of *Practice*, which is the Design of the following Treatise.

I know there are many large, and pompous Books, written on this Subject: In a great Part of which, the Authors have been free enough of their *Examples*, but too sparing of their *Demonstrations*; and some few others, have demonstrated much more, than they have shewn the Use of; nor are there those wanting, who have justly mix'd both these together.

In this little Book, I would hope that the *Mathematical* Reader, may find both as much *Demonstration*, and as much *Practice*, as may enable him to perform any Problem whatsoever, relating to these Matters, in which the Stress of the Solution is to lie

lie upon *Geometry*, and not upon bare *Delineation*. The Art of *Colouring*, is quite another thing, and for is that of *neat and curibus Drawing*; both which may be in great Perfection without the rigorous *Mathematical Part*, as the *Mathematical Part* may be without them. 'Tis this *Latter* that is my Business here in this *Treatise*, which, if it serves in any Measure either to entertain those that are knowing this Way, or to inform those that are not; I have obtained my End.

E R R A T A

Page 57. Line 26. Read *Prop. IX.* In the Corollaries of *Prop. XF* and *XII*, the Figures referr'd to, will direct the Reader when to read *N*, and when *O*. The Point *O* is intended to be in the Middle, p. 65. l. 13. Fig. 12. From p. 65. to *Prop. IV.* p. 82. the No. of the Scheme, is unity less than it should be. From p. 82, to 90, the No. is right; and from p. 90 to the End of the Book, is defective as before. Pag. 148. l. 21. dele; And the Height of the Eye. Pag. 149. l. 7. dele by. Pag. *ibid.* read l. 19, 20, 21, 22, thus; The Height of the Eye which is supposed to be unknown, we will denote by the Line *H*. But what is chiefly wanted, is that particular Distance of the Eye, &c. Pag. *ibid.* at the End of l. 27. after the Words, of *N* to *M* add, viz. For the Eye's Height will easily be found, when this Distance is once determin'd: Just as 'tis at Coroll. 1. *Prop. V.* Pag. 162. Line 2. for by that, read that by.

DEFT.

DEFINITIONS.

DEFINITION. I.

PERSPECTIVE is an ART which teacheth how to delineate the true Appearances of Objects, upon any Superficies, for any given *Distance* and *Height* of the Eye.

D E F. II.

The *Perspective Table*, or Plane, is that, whereon the Picture of the Object is form'd, according to perspective *Rules*.

D E F. III.

The Geometrical, or ground Plane, is that whereon the *Perspective Table* is sup-
posed to stand.

D E F. IV.

The *Height of the Eye*, is a Perpendi-
cular let fall from it, to the Ground Plane.

B.

D E F. V.

DEFINITION V.

The *Distance* of the Eye is a Perpendicular let fall from it, to the Perspective Table.

DEF. VI.

The common Section of the Perspective Table, with the *Ground Plane*, I shall call the *Ground Line* (or *Section*.)

DEF. VII.

The *Horizontal Line*, is a Line in the Table, Parallel to the *Section* or *Ground Line*, and of the Height of the Eye above it.

DEF. VIII.

The *Principal Ray*, is the Line let fall from the Eye Perpendicular to the Table, and therefore is equal to the *Distance of the Eye from the Table*.

DEF. IX.

The *Distance* of any Point in the ground Plane, from the Table, is a Perpendicular let fall from the Point, to the *ground Line*.

DEF. X.

DEFINITION X.

Direct Parallel Lines, are such as cut the ground Line or Section at Right Angles.

D E F. XI.

Oblique Parallels, are such as are drawn cutting the ground Line or Section, at any oblique Angle whatsoever.

D E F. XII.

Transverse Lines, are those which cut the *Direct* Lines at Right Angles.

D E F. XIII.

Radial Lines, I call such as run up from any Points in the ground Line, to any Perspective *Focus*, whether the Point of Sight, or *accidental* Point, &c.

D E F. XIV.

By the *Point of Sight*, is understood that Point in the Table, in which all the *direct* Parallels seem to concur. How it is determined, we shall see afterwards.

B 2 D E F. XV.

DEFINITION. XV.

The *Accidental Point*, is a Point which bears the same Relation to such Parallels as are *oblique* to the Ground Line, as the *Point of Sight* does to those which are *perpendicular* to it: That is, as the *Point of Sight* is that in which all the *direct* Parallels seem to concur; so in like manner, the *accidental Point*, is that in which any *oblique* Parallels do appear to the Eye to meet and unite. So that tho' (strictly speaking) there be but one *Point of Sight*; yet, there are innumerable *accidental Points*, even as many, as there are different Degrees of *Obliquity*, in which the *ground Line* or *Section*, may be cut by the foremention'd *oblique* Parallels.

D E F. XVI.

The *Point of Distance*, is a Point in the Horizontal Line of the Table, determin'd therein, by laying off from the *Point of Sight*, either way, the Eyes Distance from the Table.

D E F. XVII.

A *Point of Incidence*, is a Point in the ground Line, determin'd by a *Perpendicular*, let fall from any Point in the *ground Plane*, thereto.

D E F. XVII

DEFINITION XVIII.

The Perspective of any Point, is there, where a *visual* Line drawn from the Eye intersects the Table; or 'tis the Intersection of the Plane of the Table, by a *visual* Line drawn to that Point:

D E F. XIX.

The Perspective of a *Line* either *Straight* or *Curve*, is the common Section of the Plane of the Table, and the *visual* Superficies (whether Plain or Curve) whose Basis is the aforesaid Line.

D E F. XX.

The Perspective of any *Plane Figure*, *Rectilineal*, or *Curvilineal*, is the Section of the Cone or Pyramid (whose Vertex is the Eye, and Basis, the Figure propos'd) by the Plane of the Table,

D E F. XXI.

The Perspective of a *solid* Figure, is the Aggregate of the Perspectives of all the Planes (whereof that Solid is composed) aptly and truly set together, upon the Plane of the Table,

DEF. XXII,

DEFINITION XXII

The *Optick Angle*, under which any Object appears, is that which is comprehended under 2 Lines drawn from the Center of the Eye, to the two Extremities thereof.

According as *this Angle* is bigger or less, so we commonly suppose things to appear bigger or less to us. And it is most certainly true, that they do so, in Varieties of Cases: But that they do so in all Cases, is as certainly false. As generally as the Rule passes amongst the *Opticians*, it is not universally true, that an Object which is seen under a *bigger Angle* than another Object is, does therefore *appear bigger* to the Eye. And this will be sufficiently made out, by the following Demonstration, which is *Experimental* and *Ocular*.

Suppose that there were placed at A, (Fig. 2.) the Eye of a Spectatour, in some *long Room* or *Walk*; the Eyes Height being A B, and B K the *Walk* or Ground Plane, parallel to the Horizon. Let the Height of the Spectatours Eye, *viz.* A B, be laid off in the Ground, from his *Foot*. at B, to N; so that $B N = B A$. Then since $A B N = 90^\circ$, 'tis plain that $B A N = B N A = 45^\circ$. Therefore $B A N > N A I$, $N A V$, $N A K$,

NAK, or any other Angle, comprehended between the Horizon BK, and a Ray drawn from the Eye at A. But it is plain in *Fact and Experience*, that the Distance BN shall not appear *equal*, or *bigger*, but *less* than NI, NV, NK, &c. in the Horizontal Line: And yet BN is seen under a bigger Angle than any of all the Distances, NI, &c.

Therefore it is not universally and absolutely true, that every Object which is seen under a bigger Angle than another, does therefore *appear bigger* than that other Object does. Q. E. D.

COROLLARY

Therefore neither is it universally true, *That Objects must be seen under equal Angles, in order to their appearing equally Big.*

For here, the Distance NI *ex. gr.* appears as big to the Eye, as BN does, and and yet the Angle NAI, is much *less* than BAN. Nay, (according as the Distance is taken) NI shall appear prodigiously bigger than BN; tho' the Angle (as is observed) be still demonstrably *less*: So that there is no Doubt of the Truth of the Corollary.

And therefore I must say farther, That since this Rule (of Objects appearing equally

Bigg, which are seen under equal Angles) is so frequently made use of in most Books of *direct Opticks*, and so many things are grounded upon it, as we find there are; an accurate Enquiry ought to be made, in what Cases it holds true, and what not. In the mean time, I shall offer a few things concerning it, which Reason and Observation together, render me pretty well satisfied of the Truth of.

1. That the Rule holds true, when it speaks of Spaces or Intervals, taken in any Line, on each side a *Perpendicular*, let fall from the Eye to the said Line, and equally removed from that *Perpendicular*.

2. That it is true likewise, when it speaks of Lines *Parallel to each other*, and which lie in such a Position to the Spectatour, that a Line drawn *directly forwards* from his Foot, crosses those *Parallels* at Right Angles. Let the Parts view'd, lie in *equal Circumstances of Distance*, from this Cross Line; and then, such Segments of these *Parallels* as are intercepted between *visual Rayes making equal Angles*, will without doubt appear of *equal Bigness*, when survey'd with a free Cast of the naked Eye.

3. That the Rule is always false, when it is apply'd to Spaces, taken in one and the same Right Line, one and the same way; by which I mean, only their being taken on
one

one and the same Side of a Perpendicular, less fall from the Eye. Thus for Example, it was in the Case of the Demonstration produced: And it would be the same, if we were to look in *Breadth* or *Height*, as well as in *Length*.

I do not exclude other Cases, besides these which I have mention'd, from being Instances either of the Truth or Falsehood of the Rule. These are only such, as are the most common, and the most easie to be try'd.

SCHOL. I. *To the Preceding Definitions.*

Tho' the Perspective Table may be plac'd in *various Positions*, with Respect to the Eye, or *Ground Plane*, whereon it stands, yet it is commonly imagined to be *perpendicular* to the *Ground Plane*; this Position being of all others, the most *ready and familiar to us*. Tho' we shall shew in the ensuing Part of this Treatise, how the Rules of drawing Pieces of Perspective, upon Tables *Perpendicular* to the Horizon, may be accommodated to Tables, *in any other given Position* whatsoever.

In like manner, tho' we may conceive the Appearances of Objects, to be delineated upon Curve Superficies, whether *Convex* or *Concave*, as well as on *flat* or *plain* Superficies;

Superficies; yet for the same Reason as before, the Perspective Table is most commonly taken to be a *Plane*.

S C H O L. II.

Hitherto we have only mention'd such Lines (amongst the Definitions) as lie in the *Horizontal*, or Ground Plane *beneath* the Eye. But as we may conceive an infinite Number of other Planes, *ex. gr.* Horizontal ones *above* the Eye, *Direct*, *Declining*, *Inclining*; and such as do both *Decline*, and *Recline*, or *Incline* together; so the various Sorts of Lines which may be drawn in these Planes, are also to be considered, and will all fall under the *General Rules*, hereafter to be deliver'd.

EXPLANATION to the foregoing Definitions.

Fig. 1. W K L the Geometrical or Ground Plane.

A B D C the Perspective Plane or Table.

H the Place of the Eye.

P H the *Height* of the Eye, = E F in the *Table*.

H E the principal Ray, or *Distance* of the Eye from the Table, which is = P F in the *Ground Plane*.

D Y,

DY, Cn, OT, yS, *direct* Lines.

VW, XZ, Two *transverse* Lines.

E the Point of Sight.

mR, rs, tU, MQ, oblique Parallels.

Dt Mn OF ymr dC, the *Section* or ground Line.

Pn, a Line from P parallel to the oblique Lines tU, MQ.

Pd Parallel to the oblique Lines mR, rs, fAGE gBL, the *Horizontal* Line.

HG, a Line from the Eye parallel to Pn.

Hg, a Line from the Eye parallel to Pd.

G, g, the *accidental* Points, relating to the oblique Parallels

tU, MQ, and mR, rs, respectively.

P R O P. I.

The farther Parallel Lines are produced from the Sight, the nearer they seem to approach to each other; provided the Eye be placed any where, between the said Parallels.

This is true, whether the Eye, be in the same Plane, with the Parallels proposed, or whether it be raised above, or depressed below them.

1. Let the Eye at A, be placed in the same Plane, with the Parallels BK, RM.
(Fig. 2.)

C O N.

CONSTRUCTION.

Draw DC, LI, MK, &c. Perpendicular, as also AQ Parallel to the Lines BK, RM.

DEMONSTRATION.

The \triangle s LAI, MAK (whose common Vertex is A) have the Base LI = the Base MK; but Perpendicular AQ > Perpendicular AP (by Hypoth.) therefore Angle MAK < LAI, therefore MK appears < than LI, and the Parallels appear *nearer* to each other in the Points M, K, than in L, I. Q. E. D.

2. Let the Eye at B, be placed *above* or *below* the Plane, in which the Parallels AH, DK, are drawn. (Fig. 3.)

CONSTRUCTION.

Cross the Parallels, with the Perpendicular Lines AD, EG, HK, &c. From B, let fall BC perpendicular to the ground Plane, and carry out the *visual* Rayes BE, BG, BH, BK. From C, draw CI Parallel to AK, DK; and tho' the Lines BC and CI, conceive a Plane to pass, whose
common

common Section, with the *Ground Plane*, will be that same Line *CI*; and with the two *visual Planes*, will be *BF* and *BI*. *Lastly*, join the Points *C* and *K*, with a Right Line.

DEMONSTRATION.

Because (*Constr.*) *BC* is Perpendicular to the Plane *ADHK*, therefore the \triangle^{1s} *BCI*, *BCK* are Rectangular at *C*. farther, since (*Constr.*) *IK* is perpendicular to *CI*, therefore the \triangle^1 *CIK* is also rectangular at *I*.

Therefore, $BK^2 = CK^2 + BC^2 = CI^2 + IK^2 + BC^2 = IK^2 + BI^2$. So that $BK^2 = IK^2 + BI^2$. Therefore the Angle *BIK* is a Right one, and therefore *BIH* is a Right one. After the same manner, it may be demonstrated, that the Angles *BFG*, *BFE* are Right ones. Therefore, in the Rectangular Triangles, *BIH*, *BFE*, because the Base *HI* = *EF* (*Hypoth.*) and *BI* > *BF* (for by *Hypoth.* *CI* > *CF*) therefore shall Angle *EBF* > *HBI*.

For the same Reason, *GBF* > *KBI*. Therefore, *GBE* > *KBH*. Therefore the Parallels seem nearer to each other, in the Points *H*, *K*, than in *E*, *G*. *Q. E. D.*

COROL.

COROLLARY I.

It is certain therefore, that Lines which are *really* Parallel, cannot be seen *Parallel*.

For to be seen *Parallel*, they must appear *Equidistant* in all their Parts; whereas we are assur'd by the foregoing Demonstration, that they seem continually to approach each other: That is, they appear *Converging*.

COROL. II.

Parallel Lines indefinitely produc'd, will appear to the Eye, to meet in a Point; because the Optick Angle, subtended by the Interval or Distance between them, at that Indefinite Prolongation, will become Insensible, or of no Quantity in a Physical Sense.

PROP. II.

The Rate at which Parallels seem to converge, is determin'd by the *Reciprocal Proportion of the Tangents of the Optick Angles, to the Perpendicular Distances of the Eye from the said Parallels.*

I. If the Eye be in the *same Plane*:
(Fig. 2.) Let the Parallels be R M, A Q,
and

and the Eye at A, and the *Rays* AM, AL, AQ, cut the Line DC, in F, E, O, respectively.

From Similar \triangle^{ls} AMQ, AFO,

AO : AQ :: FO : MQ,

From Similar \triangle^{ls} ALP, AEO,

AO : AP :: EO : LP,

Therefore AQ : AP :: EO : FO;

But EO : FO :: T, EAO : T, FAO.

Therefore AQ : AP :: T EAO : T, FAO.

Q. E. D.

2. If the Eye be out of the Plane, (Fig. 3.) let the Eye be at B, the Parallels AH, CI, and the *visual* Rayes, as before.

In the Rectangular \triangle^{ls} BIH,

BI : HI :: RA : T, HBI,

In the Rectangular \triangle^{ls} BFE,

BF : EF :: RA : T, EBF,

Therefore BI : BF :: T, EBF : T, HBI.

Q. E. D.

COROL. I.

Hence we see how the visible Magnitude of an Object increases or decreases, in its various Approaches to, or Removes from the Eye, viz. thus, That the apparent Diameters, are reciprocally as the Distances from the Eye.

COROL. II.

COROL II.

The Eye, in the same Position, looking at the same Object, removed to various Distances, EF, HI, there is a *less Proportion*, between the Tangents of the Optick Angles, when the Eye is placed *above* at B, than when it is *below* at C.

CONSTRUCTION.

Draw FL Parallel BI.

DEMONSTRATION.

$T, ECF : T, HCI :: IC : CF$, by No. 1.
 $IC : CF :: IB : LF$, by Similar \triangle^{ls} BIC, LFC.

Therefore, $T, ECF : T, HCI :: IB : LF$.

But $T, EBF : T, HBI :: IB : BF$, by No. 2.

And $IB : LF > IB : BF$. Therefore,

$T, ECF : T, HCI > T, EBF : T, HBI$.

Q. E. D.

COROLLARY II.

Parallel Lines seem to *converge faster*, to an Eye posited in the *same Plane* with them, than to an Eye raised *above*, or depressed *below* that Plane.

SCHOL.

S C H O L.

In arguing here upon the Appearances of *Parallels*, we have taken the Truth of the old Maxim for granted, *That a Space seen under a less Angle, appears less, and under a bigger, greater.* And I believe the Case is so plain, that there will be little Dispute about the Truth of it here.

P R O P O S I T I O N . I I I .

If the Eye be seated any where without the Parallels, they will seem to go farther from each other (or their Intervals to widen) to a certain Term of Distance ; and after that, continually to approach each other.

C O N S T R U C T I O N .

Let the *Parallels* LA, KC, (Fig. 4.) whose *Distance* EG, is bisected in F, and and FQ drawn parallel to them. Let the Eye be at D, in the Line EG produced. Upon the Center F, with the Radius FD, strike the Circle DIH. On each other Centers, as M and B, taken at Liberty in the Line RF, and with the Radii MD, BD, strike other Circles; the former of which, imagine to cut the *Parallels* in the Points L, K; and the latter, in N, P. It is plain, that the Lines, LK, NP, shall each be
C
equal

equal to E G. For since the Centers M, B, are taken in the Right Line F R, which Line *perpendicularly* bisects E G, in the Point F; it is evident that the Lines, *joyning the Intersections* of these Circles, with the Parallels, *viz.* I H, L K, N P, shall be so many *equal Chords*, in these several Circles.

Draw the Lines D I, D H, D N, D P, D L, D K.

DEMONSTRATION.

Because the Angle D F B is \angle r, therefore in the \triangle^1 D F B, the Side D B $>$ D F. So in the \triangle^1 D F M, for the same Reason, D M $>$ D F. Therefore the Circles, whose Centers are B and M, and Radii D B, D M, are greater than the Circle, whose Center is F, and Radius D F. Since therefore the Chord I H = N P = L K, and the Circle D I H is the least of all the Circles; also the Angle I D H shall $>$ N D P, or L D K. And therefore the Parallels appear farthest asunder in I H, and from that *Limit* seem to approach to each other, both ways, *viz.* on one side towards N P, and on the other towards L K. Q. E. D.

S C H O L.

As it has been shewn in some Instances, that Lines which are *truly and strictly* parallel, will seem not to be Parallel; so it may

may also be shewn *how*, and in *what Circumstances*, Lines which are *really not Parallel*, may yet appear either *Parallel*, or else as *Parallels*. For we must take Care here, not to confound together two Notions, which in the Nature of Things are very different, *viz.* being seen as *Parallels*, and being seen *Parallel*. For two Lines to be seen *Parallel*, is for those Lines to appear equally distant in all their Parts (as was hinted before, at Cor. 1. Prop. 1.) But for two Lines to be seen as *Parallels*, is for those Lines to appear, *after the manner of Parallels*, or to appear as *Parallels use to appear*; that is, to seem inclining and converging towards each other, after the manner that such Lines seem to do. Thus 'tis demonstrable, that *two Right Lines, which are not parallel to each other*, may yet appear to the Eye, (disposed at a *certain Distance and Position*) as *Parallel Lines use to appear*.

For if those Lines be produced, till they concur, and the Angle contained between them, be bisected, and the bisecting Line be crossed at Right Angles, by two Right Lines, terminating on each Hand, in the converging Lines first given; then a Circle being describ'd, about the Trapezium thus form'd, and two Lines to touch this Circle, being drawn from the Point, where the Lines at first produced, met one another; and lastly, the two Points of

Contact being joined by a strait Line: The Distance between the *Point of Concourse* aforementioned, and the *Point* where the *Line joining the Contacts*; crosses the *bisecting Line*, is the *Diameter* of a *Semi-Circle*, which will be the *Locus* requir'd; or such, that the *Eye* being placed in any *Point* thereof, the given converging *Lines* shall appear to it, as *Parallels* would appear. The *Analysis* of this *Problem*, evinces, That the *Locus* is a *Circle*, as also how it is to be constructed; but as the thing it self is not essential to my *Purpose*, so neither is this a proper *Place* for such *Enquiries*.

Again, it might be shewn, in like manner, how two *Lines* not parallel, one being a *Right Line*, and the other a *Curve*; may, notwithstanding, appear *Parallel*, or equidistant in all their *Parts*. For if a *strait Line* be drawn in a *Plane*, and some fixed *Point* taken therein, as a *Pole* or *Center*, about which, the said *Line* revolves, keeping still in the said *Plane*, while, at the same time, another *Right Line* making any *oblique Angle*, with the *Plane*, revolves about the same *Point*, describing thereby a *Conical Surface*: also if a *Second Plane* be conceived to be drawn, either *Perpendicular* or *Oblique* to the former *Plane*, by which Means, some one or other of the *Conick Sections* is produced, then 'tis demonstrable, that to the *Eye*, posited
in

in the *Pole* (which is also the *Vertex* of the *Cone*) all those *unequal Intervals*, contain'd between the *Conick Curve*, and that *Right Line*, which is the common Section of the two aforesaid Planes, will appear of equal *Bigness*, provided the common Sections of the Planes of *Visual Rayes*, with the *Second Plane* abovementiond, be all *Parallel* one to another. N. B. When the *Second Plane* is *perpendicular* to the *First*, the *Curve* form'd, will be an *Hyperbola*; when *Oblique*, a *Parabola*, or *Ellipsis*.

P R O P. IV,

All Planes seated above the Eye, seem to sink the more downwards, the further they are produced: Those that are below the Eye, seem to rise upwards; those on the Right Hand to approach to the Left, and those on the Left, to the Right,

CONSTRUCTION.

Let the Eye be A, (*Fig. 2.*) its Height A B, a Plane above the Eye R M, a Plane below the Eye B K, the Table D C. Draw the Rayes A L, A M, A I, A K: Then shall the Points L, M, appear in E, F, and the Points I, K, in G, H.

D E M O N S T R A T I O N.

In the *Rectangle Triangles* K A L, R A M, whose Base B A is Common, the Perpen-
C 3 dicular

dicular, $RM > RL$, therefore the Angle $RAM > RAL$, therefore AM falls *with out* the Line AL , and therefore cuts the Table DC in a Point B *lower* than E .

In like Manner, it will be proved that the Point K appears *Higher* in the Table than I .

And so it may be prov'd by the same way of arguing, That Planes lying on the *right* Hand of the Eye, seem to approach nearer and nearer to the *Left*, as those also on the *Left*, to approach to the *Right*.

For we need only to suppose the Eye A , to be plac'd between two Planes, as BK on the *Right*, and RM on the *Left*. Therefore, *Q. E. D.*

SCHOL.

The Truth of the Proposition may otherwise thus appear. Since any visible Point as M , appears not to the Eye in the same Place that it really is in, but in some other Place in the same Ray AM , nearer, as at N ; so likewise, since the Point T , is not seen in T , but somewhere nearer, as at n : For this Reason, the Space TM shall appear in nN , that is, *falling downwards*.

And for the same Reason, the Space IK shall appear in sS , *rising upwards* towards the Eye.

But

But 'tis to be observ'd, that as the Points L and I , are not seen there, but somewhere nearer in the same Rayes AL , and AI ; so consequently the Space LM , cannot appear in LN , nor IK in IS , and therefore the Representation of the Planes LM , IK , cannot be the Lines $Ln Nn$, $Is Ss$, as they are here drawn from the Points L and I : Because, I say, the Points L and I , being not seen where they are, but nearer to the Eye A ; the Lines $Ln Nn$, $Is Ss$, cannot begin at the Points L and I , but at some other Points between them and the Eye A . As for the *Species* of these Lines, it's manifest they cannot be *strait* Lines, but *Curves*, approaching continually nearer and nearer to the Line AQ produc'd.

Which Line AQ , will be as a common *Asymptote* to them.

The *Nature* of these Lines is to be determined by *Observation* and *Experiment*; namely, when it shall be determined at what Distances the Points T , M , do appear in the Rayes AM , AT , from their true and real Places; that is, how far the Points n , N , &c. are from the Points T , M , &c.

From the Proposition before demonstrated, we may see the Reason of several Appearances, which are very common.

C O R O L. I.

The *Floors and Pavements* of Buildings (especially those that are very long) seem to rise upwards, towards the Eye of the Spectatour, that enters them.

C O R O L. II.

For which Reason, in *Churches, ex. gr.* the Pavement, in going from the *Door*, towards the *Altar*, need not be raised above the Level, so that a Person should continually ascend in approaching towards the *Latter*, from the *Former*: Because, besides that there is already an Ascent, which proceeds from the Principles of *Opticks*; which therefore ought not to be made yet more considerable, by an actual Elevation of the *Floor*; there would be this farther Inconvenience, in raising it above the Level, *viz.* That the Orders of the *remoter Columns*, being therefore necessarily shorter than those nearer the Eye, they would be so immoderately shortned in the *Appearance*, as to offend the Spectator's Eye very much at his Entrance.

C O R O L. III.

The *Roofs and Cielings* of Buildings, appear gradually to sink down towards the Eye.

C O R O L. IV.

C O R O L. IV.

And therefore, any *Roof*, or *Consignation* ought to be so much the Higher, by how much, the *Area* which lies under, extends it self farther in *Length*.

For otherwise, at a considerable Distance, it would seem to hang down upon the very Ground it self.

C O R O L. V.

Long Rows of *Columns* or *Pilasters*, *Trees*, *Walls*, and the *Sides* of *Buildings*, contract themselves to the Eye, and seem to grow narrower and narrower.

C O R O L. VI.

And for this Reason, in order to make *Prospects* of this Kind truly pleasing and agreeable; Care should be taken, that the *Breadth* or *Wideness* of them, be duly proportion'd to the *Length* they are design'd to be of.

S C H O L.

A Man may at any time, experiment the Truth of the foregoing Corollaries, in a long *Portico* or *Piazza*, adorn'd with Orders

ders of Pillars. There he may see, how the *Pavement* seems to rise, the *Roof* to sink down towards the Eye, and the *Side-walls* to incline to each other; and all verging to a Point: which Phænomenon was most accurately describ'd by the *Philosophical Poet*, in those excellent Lines.

*Porticus aquali quamvis est denique ductu
Stansque in perpetuum paribus suffulta Co-
lumnis,
Longa tamen in parte absurra cum tota vi-
detur,
Paulatim trahit angusti fastigia Coni;
Tecta Solo jungens, atque omnia Dextera
Lævis,
Donicum (or Donec) in obscurum Coni con-
duxit Acumen.*

Lucret. Lib. 4.

C O R O L. VII.

The *Capitals* of Pillars appear inclining downwards, and the *Pedestals* rising upwards.

C O R O L. VIII.

The *Horizon* appears higher, than it really is. For, because of the immoderate Distance between it and the Eye of the Spectator; it seems to be of an equal Height

Height with the Eye it self. And therefore every Spectator has a *different optical Horizon*, according to the different Altitude of his Eye above the Plane of the *real sensible Horizon* (which is a *Tangent* to the Surface of the Earth, in that Point where the Person stands.)

C O R O L. IX.

For the same Reason, the Convex Surface of the *Sea*, to an Eye placed thereon, appears differently *Protuberant* and *Curv'd* from what it is in it self.

C O R O L. X.

It follows likewise, that if a Row of Columns (*ex. gr.*) all equal in Height, and Perpendicular to the Horizon, were dispos'd in Order *beneath* the Eye; those which are the *remotest*, would appear to be *lifted up higher*, in Proportion, than the rest.

But if they were disposed *above* the Eye, those which are the *remotest*, would seem to be more *sunk or depressed*, than the *nearer* ones.

For by the *Proposition*, this is true of any Points (in these Magnitudes) which are terminated in the same Horizontal Line; therefore, it is true of all Points in them, terminated by Horizontal Lines; that is, of the whole Magnitudes themselves.

S C H O L.

S C H O L.

From this *last Corollary*, arises another Consideration which deserves Regard, *viz.* That *Superficies*, which are exactly plain and level to the *Horizon*, plac'd ex. gr. above the Eye, must necessarily appear sunk in and hollowed.

This infers the Reason and Use of those *Scamilli*, whereof *Vitruvius* speaks, as a Remedy to prevent some unpleasing Appearances, in a piece of *Architecture*. *Stylobatam ita oportet exaquari uti habeat per medium adjectionem per Scamillos impares; si enim ad Libellam dirigatur, Alveolatus oculo videbitur. Vitruv. Lib. 3. Cap. 3.*

The same Consideration is likewise of use in the shaping of *Images* and *Statues*, which are to be plac'd at considerable Heights above the Eye. For a Figure which shews all the exact Symmetry and Proportion, in the World, to the Eye, at one Elevation or Distance, will perhaps, lose all those Charms, and become downright ugly at another. So that in those Cases, *Art* is to consult and see, what is to be *Added* or *Taken away*; that the great Ends of Beauty and Pleasure may be provided for,

for, according to the *Nature and Conditions* of the Place from which an Object is to be viewed.

Alia enim ad manum Species esse videtur, alia in excelsa, non eadem in concluso, dissimilis in aperto; in quibus magni Judicii est Opera, quid tandem faciendum sit. Vitruv. Lib. 6. Cap. 2.

And again, *Cum ergo quæ sunt vera, falsa videantur, & nonnulla aliter quam sunt oculis probentur; non puto oportere esse dubium, quin ad Locorum Naturas aut Necessitates, Detractiones aut Additiones fieri debeant: sed ita ut nihil in his operibus desideretur.*

It was owing not only to Knowledge in *Sculpture*, but to Skill in *Proportions*, and especially to the Knowledge of *Optical* Appearances, and the Reasons of them; that the celebrated *Phidias*, at once surpriz'd all the People of *Athens*, and triumphed over *Alcámenes*, who was his Rival, for Fame and Glory in the Art of Carving.

The *STORY* we have in *Tzetzes, Var. Hister. Chil. 8. Hist. 193.*

‘ **T**Hough these Persons were both
 ‘ of them excellent *Statuaries*, yet
 ‘ *Alcámenes* understood only the *Mechanick*
 ‘ servile Part of his Art; whereas *Phidias*
 ‘ being

' being well seen in *Geometry* and *Perspe-*
 ' *ctive*, knew how to render his Work
 ' compleat by the Rules of those Sciences.
 ' Now, the *Athenians* having appointed a
 ' Statue of *Minerva* to be set up in the
 ' Market Place: Each of these Artists,
 ' was order'd to imploy his best Skill in
 ' the making of one. Accordingly, *Alca-*
 ' *menes* made a Statue of such charming
 ' Beauty, to an Eye which view'd it at
 ' a *small Distance*; that all the People at
 ' first Sight, adjudged him the Victory.
 ' And they thought themselves still more
 ' in the Right, when *Phidias's* Work ap-
 ' pear'd. For, he considering at what
 ' *Height* the Statue was to be plac'd, had
 ' shap'd it accordingly; making the Coun-
 ' tenance horribly distorted, and all the
 ' Limbs so disproportion'd, that it look'd
 ' more like the Figure of a Devil than a
 ' Goddess. And the *Mob* (who never judge
 ' by Reason, and the Rules of Art, but by
 ' present Sense) were well-dispos'd to have
 ' made him sensible of their Resentments,
 ' upon the Score of the Affront they
 ' thought was offer'd to *Pallas*; in making
 ' such a filthy Thing to represent her:
 ' However, they houted him, and cried
 ' up *Alcamenes* for an Artist beyond Com-
 ' parison. And thus Matters stood (*Phidias*
 ' enduring the Persecution of the ignorant
 ' Rabble)

‘Rabble) till both the Statues came to be
 ‘set up at the *appointed Height*. But then the
 ‘Scene was quickly chang’d. All the soft
 ‘Strokes and Graces of *Alcamenes’s* Image,
 ‘quite disappear’d; as on the other Hand,
 ‘did the rough and barbarous Features of
 ‘that made by *Phidias*. So that now (both
 ‘being view’d at the *proper Distance*) the
 ‘*Former* appear’d ugly, and the *Latter*,
 ‘exquisitely fine and beautiful: And so
 ‘*Phidias*, (besides the *Prize*) went off with
 ‘as much *Praise*, as before he had *Con-*
 ‘tempt, from the Common People.

Such Trials of Skill are sometimes seen
 in *other Arts* besides Sculpture; and there
 are more *Alcamenes’s*, and *Phidias’s*, be-
 sides those who contended at *Athens*.

PROP. V. THOR. V.

*If the Object be a plane Figure, seated in a
 Position parallel to the Table; its Per-
 spective will be a plane Figure similar
 thereto. (The Picture and the Original,
 will be like each other.)*

Tho’ this may easily be conceiv’d, for
 the Section of the *visual Pyramid* or *Cone*,
 by a Plane parallel to its Base; I shall, not-
 withstanding, demonstrate it *in Form*, for
 the

the sake of those who may desire to see a strict Proof, for all the Conclusions advanced to them, in this Science.

CONSTRUCTION.

The Object (Fig. 5.) being the Figure $DEFL$; from the Eye at A , draw to the several Angles thereof, the visual Rays AD , AE , &c. by which Means the *optick Pyramid*, $ADEFL$ is formed, and is also cut by the Plane of the Table $NRKS$ Parallel to $DEFL$, the Section produced being $BCGI$. Draw LE , and IC .

DEMONSTRATION.

Because the two parallel Planes are cut by the Plane AEF ; the common Sections FE , GC , shall be parallel. Therefore the \triangle^{ls} AGC , AFE , are Similar. So likewise, are AGI and AFL , and for the same Reason ACI and AEL .

Wherefore, $AG : AF :: GC : FE$.

Also, $AG : AF :: GI : FL$.

Therefore, $GC : FE :: GI : FL$.

Also, $AG : AF :: AC : AE$.

And $AC : AE :: CI : EL$.

Therefore, $AG : AF :: CI : EL$.

Therefore, $CI : EL :: GC : FE ::$

$GI : FL$. Wherefore, since the Sides of
the

the \triangle^{ls} CIG, ELF, are thus proportional, it follows, that they are *Equiangular*.

And thus may all the rest of the \triangle^{ls} , whereof the *Original*, and the *Image* are composed, be shewn to be Similar.

Therefore the *polygonal* Figures themselves are so, Q. E. D.

C O R O L. I.

The Object being any *Rectilineal* plane Figure, its Perspective is *Dissimilar* to it, when its Position is not Parallel to the Plane of the Table.

But if a *Circle*; its Perspective may be *Similar* to it, that it may be a Circle in some certain Position; tho' it does not lie Parallel to the Table, because the *visual Cone* may be cut *subcontrarily*, by the Plane of the Table.

Thus (Fig. 6.) suppose the Object DE a *Circle*, the Table GD, the Eyes Distance BD. We may determine from these *Data*, a proper *Height* of the Eye as FB, so that the visual Cone DFE may be cut *subcontrarily* in CD, and consequently the *Perspective* be a Circle too. Take BA = BD, and then bisect the Line AE in H, so that

$$AH \text{ or } EH = \frac{DE + 2 BD}{2}; \text{ on the Center}$$

ter H, and with the Radius AH strike a Circle, cutting a Perpendicular from the Point B, in F; which will be the *Place* of the Eye *sought*, and consequently FB its *Height*. Joyn AF. The Angle AFE = \perp (because of the *Circle*) therefore since FB Perpendicular to AE, also the Angle AFB shall = BEF. But because AB = BD (*Construct.*) therefore BFD = BFA, therefore BFD = BEF. But BF parallel to GD, therefore BFD = CDF, therefore DEF = CDF, therefore the \triangle^1 FCD is Similar to the \triangle^1 FDE. Wherefore the Cone is cut *subcontrarily*, and the Perspective CD is a Circle.
Q. E. D.

COROLLARY II.

Hence may be found such a Distance, of the *Object* or *Eye*, from the Table, that the Perspective shall not only be *Similar*, but also in any given Proportion to the *Original*. *Ex. Gr.* If the Eyes Distance being given, such a Distance between the *Object* and the *Table* were required, that DLFEZT : BIGCOV :: p : q.
Since p : q :: DLFEZT : BIGCOV :: EF^q : GC^q (because the Figures are Similar) :: AE^q : AC^q (Similar \triangle^1 s) :: AQ^q : AP^q (Similar \triangle^1 s.)

'Tis

'Tis plain, that supposing the Line APQ perpendicular to the two Planes, the Distance PQ is easily determined; *viz.*

$$PQ = \frac{\sqrt{p} - \sqrt{q}}{\sqrt{q}} \times AP.$$

C O R O L L. III.

The Object continuing in a Position parallel to the Table; whether the Eye moves nearer or farther from the Table, while the Object keeps its Distance, or the Object moves while the Eye keeps the same Distance, or the Table moves, while the Eye and Object keep their Places; in either Case, there is no Alteration, of the *Species*, of the Perspective, but only of the *Magnitude* thereof. But of these things, in another place.

P R O P. VI.

All the Conical Sections, are only the Perspective Representations of the Circular Line, of the Base, upon Tables in various Positions, to the Eye seated in the Vertex of the Cone.

It will need no Figure, to prove this Proposition to them, that know the Cone, and the several Sections of it.

For the Cone being cut by a Plane, parallel to a Plane, which coming out of the Vertex, touches the Cone in its Side; (for which is all one, meets the circular Base in *one Point* only) if the Plane of this Section, be made the Perspective Table, the Representation of the Circular Arch, will be a *Parabolical Line*.

But if the Table be parallel to a Plane, which meets not the circular Base, at all; it will be an *Ellipsis*; or if parallel to a Plane, which cuts the Basis, an *Hyperbola*.

The Reason is, because the right Lines, drawn on the Surface of the Cone, from the *Vertex*, to the several Points of the Circumference of the Base (which Lines in this Case, are our *visual projecting Rays*) do trace out upon the Planes of the several Sections (which are our *Perspective Tables*) the Conical Curves; which therefore are only so many Peices of Perspective, to an Eye posited in the Vertex. *Q. E. D.*

C O R O L. I.

One and the same Conick Section, may be the Perspective of an infinite Number of different circular Arches. For the *Geometricians* demonstrate, that any *Parabola*, may be adapted to *any Cone*; and any *Ellipsis* or *Hyperbola* (though not to any Cone, yet least)

least) to various Cones, *differing in Species.*

C O R O L. II.

Any of the Conick Sections may be, the Perspectives of each other, to the Eye (as before) plac'd in the Vertex.

Ex. Gr. Suppose a Plane cutting the Cone, and producing a Parabola. Thro' the common Section of this Plane with the circular Base, suppose an infinite Number of other Planes to pass, each cutting the Cone between its Vertex, and the Vertex of the foremention'd Parabola. Any one of this infinite Number of Planes, being taken at Liberty, for a *Table*; the Eye sees the *Parabola*, as an *Hyperbola* thereon.

And so of any of the rest. The thing is so plain, that any one, by only drawing a Cone, may abundantly satise himself of all the Particulars.

S C H O L. I.

From *this Generation of the Conick Sections*, wherein we consider them as the Perspectives of the circular Base, arises a Speculation, which is not unworthy of Notice; and that is this.

The whole Area of the *Ellipsis* in any Cone, lying all entirely *above* the Circular Basis,

Basis, or between the Vertex and it, is therefore, the Projection of that whole Basis.

But the *Hyperbola* and *Parabola*, being Curves which do not *include Space*, but run out *ad infinitum*, are projected after another Manner. In the *Parabola*, *ex. gr.* that Part which *lies above* the circular Basis, is the Projection of a *determinate Arch* of the Circle, and the remaining *infinite Portion* thereof *below* the Basis is projected from the *Complement* (of the said Arch) to the whole Circle. For the *last* projecting Rays is the Side of the Cone, parallel to the Axis of the Parabola.

In the *Hyperbola*, that Part which is *above* the Basis of the Cone, is likewise the Projection of a *determinate Arch* of the Circle, but the remaining *Infinite Portion below* the Basis, is projected, not from the Complement of the former Arch, to the whole Circle, but from the Complement thereof, *to that Arch, which is determin'd, by a Plane passing out of the Cones Vertex, parallel to that which generates the Hyperbola.* I say, the infinite Portion of the Hyperbola below the Basis of the Cone, is form'd, by projecting only that Arch (which lies between the Plane *making the Section*, and the Plane out of the *Vertex* parallel thereto) upon the Plane making the Section.

Now,

Now, as in all Projection whatsoever, either the Plane we project on, is plac'd between the Object and the Eye, or else the Object, between that Plane, and the Eye; so it has been usual to call the latter Sort of Projection, an *INVERTED PERSPECTIVE*, or a *DEFORMATION*: For in all the common *Scenographick* Representations, the Table is always plac'd between the Eye and the Object. *Now both these kinds of Projection take place, in that Generation of the Conick Sections, we are speaking of.* Nay, and both too, in the Formation of one and the same Section.

The *whole Ellipsis*, is a regular *Scenographick* Projection, the Table being between the Object and the Eye.

So likewise are those Portions of all *Parabolas and Hyperbolas*, which lie above the Basis of the Cone.

But the remaining *Infinite* Portions of those Curves, *below* the Basis, are *Inverted Perspectives* or *Deformations*; the Circular Arch, which is the Object projected, lying between the Eye, and those Parts of the Planes of these Sections, on which the Projection is made,

S C H O L. II.

Since the same general Affections which are demonstrated of Cones, whose Bases

D 4

are

are *Circles*, are applicable likewise to such Cones, whose Bases are *any of the Conick Sections*, (*per Append. de Sectionibus Pyramidum quarum Bases sunt Sectiones Conica; M. de La Hire*) 'tis plain from hence, that we may determine how, and in what Circumstances, any Conick Section, *seated in the Ground Plane*, shall become any other Conick Section whatsoever *in Perspective*: That is, what Section shall be produced, by the Plane of the Table cutting any sort of Cone, whose Base is either *Ellipsis, Parabola, or Hyperbola*,

P R O P. VII. (Fig. 7.)

It may be, that Lines, which are not parallel in the Ground Plane, may come into parallel Lines on the Table. Or, The Perspectives of Diverging Lines, may be Parallel.

Suppose the Non-parallel Lines to be PD, NE, the Eye at K, its Height KV, the Table, RSCT.

Let the Eye be so posited, that the Lines PD, NE, may lie in the visual Planes KVD, KVE; whose Intersections with the Table are AP and BN, and therefore the Representations of the aforesaid Lines to the Eye at K.

I say that AP is parallel to BN, if the Eye be so placed, For

For because KV is perpendicular to the Ground Plane, therefore the Planes KVD, KVE, are perpendicular to the Ground Plane. And because the Table RSCT is likewise Perpendicular thereto; therefore AP, and BN, the common Sections of these Planes, are perpendicular to the Ground Plane, and therefore parallel to one another. Q. E. D.

C O R O L L A R Y I.

The Trapezium PDNE is represented on the Table, by the Rectangle APBN.

C O R O L II.

Hence the vulgar Method of rectifying a deformed Object; or placing the Eye in such Manner, that a *rude and irregular Picture*, shall from a certain Point, appear regular and beautiful. For thus, the Trapezium PDNE, which may be as distorted and unshapen as one pleases, will fall on the Perspective Table in the compact Form of a Rectangle, as APBN. And therefore were the Parts of any Image, suppose a *Humane* Face) disposed up and down in the *Cells* of this Trapezium, they would appear, in an agreeable Order and Posture to the Eye, in the correspondent Cells of the Rectangle upon the Table.

C O R O L.

COROL III.

Hence it appears, that this Practice of *Deforming*, is rightly Term'd, *An Inverted sort of Perspective*.

For as in the *common* Perspective, a Rectangle *ex. gr.* lying in the Ground or *Horizontal* Plane, is projected into a Trapezium upon a *vertical* Table, placed between the Object and the Eye; so in *Deformations*, a Rectangle drawn in a *vertical* Plane, is projected into a Trapezium, upon a *Horizontal* Table, which lies farther from the Eye than the Object does.

COROL IV.

The Points D, and E, and consequently the whole Deformation, are determin'd, by drawing out the visual Rays KA, KB, till they intersect the Ground Plane in D, E, and then joyning DE.

COROL V.

Otherwise, the Lines VP, KA, and VN, KB, produced till they meet each other; meet in the same Points D and E, as before.

COROL

C O R O L. VI.

The Line DE is parallel to PN, so that the Deformation of a Rectangle, is a Trapezium, whose two opposite Sides are parallel.

P R O P. VIII. (Fig. 7.)

There is an infinite Number of Points, in which the Eye being placed, shall project Diverging Lines upon the Ground Plane, into Parallel ones on the Table; and the Locus of those Points, is easily determin'd.

Supposing all, as in the foregoing Proposition: Produce the diverging Lines DP, NE, till they cut each other in V; at which Point, erect the Perpendicular VK, which extend at Liberty. I say, the Perspectives of the Lines DP, EN, shall be parallel to each other on the Table, the Eye being placed in any Point of this infinite Perpendicular.

For taking any Point therein, as K, for the Eye's Place, and drawing out the visual Planes KPD, KNE; it's evident that these coincide with the Planes KVD, KVE which being perpendicular to the Ground Plane VDE, their common Sections with
the

the Plane of the Table, AP, BN, will be demonstrated (as before) to be parallel to each other. Therefore, &c. Q. E. D.

S C H O L.

We have in this Case, an Instance (tho' depending on another Principle) of the Truth of what was before discours'd at *Schol. Prop. 3. viz. Concerning Lines which are not parallel, appearing as Parallels.*

For since all the Points of the Lines PD, NE, appear in AP, BN, which are strictly parallel to one another; 'tis evident that the two *former* Lines are seen *as* the two *latter*, that is, *as* parallel Lines. And how that is, we have demonstrated at Prop. 1, 2, 3.

L E M. I. Fig. 8.

If the Parallels LM, GV, HN, &c. in the Base of the Triangular Prism ABLHMN, be produced at Liberty towards P, Q, R, &c. any Lines as AI, AK, drawn from the solid Angle A, to any Points as I, K, in those Parallels, shall necessarily intersect the Lines NB, VB, running up from the Points N, V, to the other solid Angle B.

DEMON.

DEMONSTRATION.

For the Line NR, *ex. gr.* being in the Plane of the Parallelogram ABHN; 'tis evident, that a Line drawn from A to a Point as I, in the Base HN, produced, shall cut the opposite Side BN in some Point as *e*, by the way ; so in the Parallelogram ABGV, the Line AK, shall cut BV, in *k* ; and so of all the rest. Therefore, &c.
Q. E. D.

L E M. II.

If an Eye were plac'd at A, it would project the Point I into *e*, or K into *k* ; or any other Points in the parallel Lines RN, VQ, into some Points of the Lines NO, VS, running up to the Angle B. And therefore, it would project the parallel Lines NR, VQ, MP, infinitely produced, into the Lines NB, VB, MB.

P R O P. IX. Fig. 8.

The Perspectives, of all Lines which are parallel one to another, and not parallel to the ground Line, do run up into one and the same Point in the Table.

This is the main and great Proposition in this Science, and is thus easily and universally

versally demonstrated, by the Help of the two foregoing Lemmata.

DEMONSTRATION.

Suppose the Parallels MP, VQ, NR, &c. I say, the Perspectives of these shall all run up into one and the same Point. By Lemma 1. the common Sections of all the Planes, AHR, AGQ, ALP, &c. with the Plane CDFE, must necessarily meet one another in the solid Angle of the Prism B.

By Lem. 2. the Plane CBDFVE is a Perspective Table to the Eye plac'd at A, the other *solid* Angle of the *same* Prism; and therefore the common Sections of the aforesaid Planes, AHR, AGQ, ALP, &c. with the Plane CDFE, will be the Perspectives of the Parallels NR, VQ, MP, and therefore these Perspectives must necessarily be the Lines NB, VB, MB, &c. all meeting in one and the same Point B.
Q. E. D.

COROL. I.

From hence again appears a Reason why in long *Rooms* and *Walls*, or Rows of *Trees* and *Pillars*, the Sides seem closer one to another towards the *farther* End, than at the Parts nearer the Place of the Eye.

COROL.

C O R O L. II.

If any Object, as I, were removed out to the Horizon, the visual Ray AI, would coincide with ABO, so would all other visual Rays coincide with each other in the same Ray AB.

C O R O L. III.

From hence it follows, that the Point to which the Perspectives of any Parallels converge, is there where a Line from the Eye parallel to those Parallels, strikes the Table. For all the visual Rayes coincide at last in the Ray AB ; which is the *common Section* or Side, of all the parallelogram Superficies ALOP, AGOQ, AHOR, &c.

Therefore since (from the Construction of the Figure) AB is parallel to each of the infinite Lines in the Base ; the Consequence is clear.

C O R O L. IV.

The Ray AB strikes the Table at right Angles, when the Lines NP, NR, &c. are perpendicular to the ground Line MN. But if MB, NR, &c, cut the ground Line obliquely ; the Angle ABC will still
equal

equal PMN or RNE, &c. that is the Ray AB, will always make the same Angle with the Line CBD in the Table (which we suppose parallel to MN) as the oblique Parallels themselves, do with the Line MN.

C O R O L. V.

If the Plane MPNR be parallel to the *Horizon*; 'tis plain that the Line CBD, will be that, which we call the *Horizontal Line*; whose Elevation above the ground Plane, is just equal to the Height of the Eye. And consequently, it will follow, that the Perspectives of all Parallels, whether *perpendicular* or *oblique* to the ground Line, do run up to some Points in the *Horizontal Line*. But if the Plane MPNR, be either *elevated* above, or *depress'd* below, the *Horizontal Plane*; then the Point where Perspectives of these Parallels will meet, will accordingly be found in the Table *above* or *below* that, which is commonly called the *Horizontal Line*.

C O R O L. VI.

If the Parallels MP, VQ, NR, &c. be at right Angles to the Line FE, then B, shall be that, which we call the *Point of Sight*;

Sight; but if the said Parallels be oblique to the Line FE, then B shall be some *Accidental Point*.

C O R O L. VII.

From what has been said, appears the Method of finding out the Points, to which the Perspectives of any Parallels, lying in the Plane of the Horizon, do converge upon the Table. *Viz.*

Draw a Line from the Foot of the Eyes Perpendicular, parallel to the Parallels propos'd, and see where it cuts the Ground Line. From that Point carry up a Perpendicular, equal to the Height of the Eye. Where that Perpendicular intersects the Horizontal Line, will be the Point sought.

Therefore, to determine the Point of Sight, is only to let fall a Line from the Eye, *perpendicularly* to the Table.

C O R O L. VIII.

When the oblique Parallels, cut the Ground Line, at an Angle of 45° ; then the *Points of Distance*, become the proper *Accidental Points*, to which the Perspectives of those Parallels converge.

E

C O R O L.

C O R O L. IX.

By how much the more obliquely, any Parallels fall on the Ground Line, by so much the farther, is the Point to which their Perspectives converge, distant in the *Horizontal Line*, from the *Point of Sight*.

C O R O L. X.

If any Angle be made *at the Eye*, equal to the Angle contained *under the Sides* of any Polygon; the Leggs comprehending that Angle, will strike the Table in those Points, to which the Perspectives of all Lines, parallel to the said Sides, will converge. Thus in an *Equilateral Triangle*; *ex. gr.* Those Points will be determined by the Leggs of an Angle of 60° : In a *Square*, by an Angle of 90° : In a regular *Pentagon*, by one of 108° ; and so in every *regular Figure* the *accidental Points* (to which the Perspectives of all Lines parallel to the Sides of that Figure, converge) are marked out by Rayes, making an Angle at the Eye, equal to the Angle of the said Polygon.

C O R O L.

C O R O L. XI.

The Perspectives of all oblique Parallels in the Horizontal Plane both *below*, and *above* the Eye, and in the two side Planes, perpendicular to the former, will all concur in one and the same Point. So that an infinite *Parallellipid*, dwindles in Perspective, into a *Pyramid*, as a *Parallelogram* does into a *Triangle*, and a *Cylinder* into a *Cone*.

C O R O L. XII.

If the Right Line, which is the Eyes Distance from the Table, be produced infinitely towards the Parts of the Eye; the same converging Lines on the *Table*, will be the Perspectives of the same Parallels in the *Ground Plane*, to the Eye seated in any Point whatsoever, of that infinite Line.

And this solves, what some have reckon'd a sort of *Paradox*, in this Science, *viz.* That the same Parallels should be projected into the self same Lines on the Table, tho' the Eye changes its Place and Distance.

S C H O L.

Since the Base of the Prism (Fig. 8.) may be as well *any sort* of Parallelogram, as a *Rectangle*; as also since the two opposite Triangular Planes, may be as well any way *inclin'd*, as stand *perpendicular* to the Plane of the Base: It follows, that the Proposition, is by *this Method* universally demonstrated, with Respect to any sort of Lines, drawn in Planes, which lie in any Manner of Position to the Table. For of what Species soever the Prism be, provided it be but a Prism, yet still the Lines MB, VB, NB, which run up to one solid Angle B, will necessarily be the Projections of the Lines MP, VQ, NR, to the Eye placed at the other solid Angle A.

P R O P. X.

The Perspective of any visible Point, is truly determin'd, by the Intersection of a Radial Line, (drawn from the Point of Incidence) and a Line connecting the Eye's Distance, set off in the Horizontal Line, with the Distance of the Point seen laid off in the Ground Line. (See Fig. 9.

C O N-

CONSTRUCTION.

Let $Cbnd$ be the Table, the Eye at A , its Height $AG = BH$, the Distance of the Point seen, D from the Ground Line $= ID = IE$, the Distance of the Eye $GH = AB = BC$; the Radial BI drawn from the Point of Incidence I , to the Point B ; the Line CE connecting the Points C, E , cutting the Radial BI in L ; the Line AD , drawn from A to the Point seen D , cutting the Table in the Point K .

I say that L is the Perspective of D .
Draw CM parallel to BI , and FI —and parallel to GH ; Join the Points GF , and lastly draw AF .

DEMONSTRATION.

Since the visual Ray AD , cuts the Table in the Point K , ; 'tis plain from thence that K is the true *Natural* Perspective of the Point D .

Also by Proposition IX, it appears, that K , the Perspective of D , must needs be found in the *Radial* BI , drawn to the Point of Sight B , from the Point of Incidence I . I shall now demonstrate, that the Point L coincides with K , the natural Perspective of the Point D .

The \triangle^{ls} DAF and DIK are Similar, for
AF \parallel BI.

Therefore DF : AF :: DI : IK.

Also the \triangle^{ls} CMB and LIE are Similar,
for CM \parallel BI.

Therefore ME : MC :: IE : IL.

Now because EI \pm DI, therefore DI
+ IF = IE + IF = IE + GH = IE + AB
= IE + AB = IE + CB = IE + MI,
therefore DF = ME.

Again, AF = BI = MC. So that the
three first Terms of the *first* Proportion,
are respectively equal to the three first of
the *latter*, therefore the fourth Terms
are respectively equal, that is IK = IL.
Therefore the Points K and L coincide.
Therefore the Perspective of the Point D,
is truly determin'd by the Intersection of
the Line CE, with the Radial BI.
Q. E. D.

C O R O L.

Hence it follows, that the Perspective
of any Point, will also be determin'd, by
the mutual Intersection, of the Lines
drawn from the *two Points of Distance*, to
those Points in the *Ground Line*, where the
Distance of the Point seen, is laid off. That
is, set off the *Eyes Distance*, both ways,
from the *Point of Sight*, in the Horizontal
Line;

Line; and the *Distance of the visible Point* from the Table, both ways, from the *Point of Incidence*, in the *Ground Line*, and connect the Points above and below *Alternately*, with right Lines; so shall the Intersection of these two Lines in the Table, be the true Perspective of the Point given.

For it may be demonstrated by the same Steps, as above, that each of these Lines of *Distance*, will intersect the *common Radial* (drawn from the Point of Incidence) in one and the same Point. Therefore, &c.

P R O P. XI.

Any Portion of a direct Line, contiguous to the Table, is to its Perspective, as the Sum of its Length, and the Eyes Distance from the Table, is to the Length of the whole correspondent Radial.

C O N S T R U C T I O N.

Let the Distance proposed be TM. (See Fig. 10.) The Perspective of the Point M is at E, some where in the Radial TH, drawn from T the Point of Incidence, by Prop. IX. Draw GY parallel to AB, and produce MT to cut GY in V. Then draw KV, from the Eye at K.

IX.

DEMONSTRATION.

Because $GY \parallel AB$, and $VT \parallel Gs$, therefore $GV = ST$, and since $KG = FS$, and the Angle $KGS = FST$, being both \angle rs, therefore $FT = KV$. Farther since $VT = GS$, $= KF$, therefore $VT = KF$. Therefore the Figure $KFVT$ is a Parallelogram, therefore $FT \parallel KV$. Therefore the \triangle ^{ls} KVM and ETM are Similar;

Therefore $TM : TE :: VM : VK$, but $VK = TF$,

Therefore $TM : TE :: VM : TF$. *Q.E.D.*

COROL. I.

When the Distance TM , coincides with SO , which runs up to the Foot of the Eyes perpendicular; then the Rule will be thus: *As the Distance seen, is to its Perspective, so is the Sum of that Distance, and the Eye's Distance from the Table, to the Height of the Eye.* For now the Length of the Radial, coincides with the Height of the Eye.

COROL.

C O R O L. II.

Equal Portions being taken, of several *direct* Lines; that which passes thro' the Foot of the Eyes perpendicular, will have its perspective Contraction, of all the *shortest*.

C O R O L. III.

Hence may be computed the Proportion, between the Perspectives of any Part of a *direct* Line contiguous to the Table, to the Eye plac'd, either at different *Heights* or different *Distances*, or different Heights and Distances both together; *viz.* By *Corol. I.* when the Line passes through the Foot of the Eye's Perpendicular; and by the *Prop.* it self, when it passes through any other Point.

C O R O L. IV.

And because the Perspective of any Part of a *direct* Line, not *contiguous* to the Table, is equal to the Difference of the Perspectives of two Parts of the same *direct* Line, *which are contiguous* to the Table; therefore, by what has been said, we can also determine the Proportion of the Perspectives,

spectives, of any Segment of a direct Line *not contiguous* to the Table, to the Eye either at different *Heights*, or *Distances*, or both together. But of this, see more by and by, at Prop. XIII.

PROPOSITION XII.

If any Line be parallel to the Ground Line, its Perspective in the Table, shall be parallel to the Ground Line also,

CONSTRUCTION.

Let the Line MN be parallel AB, (See Fig. 10.) the Ground Line; and drawing from the Eye at K, the Lines KN, KM, let us conceive the *Plane of Rayes* KNM, whose common Section with the *Plane of the Table*, suppose to be DE, which is therefore the Perspective of MN (by Def. XX,) and must now be shewn to be parallel to AB. Upon MN, erect the Plane MNXZ, perpendicular to the Ground Plane.

DEMONSTRATION.

Because the Plane NMXZ is perpendicular to the Ground Plane, therefore it is parallel to the Plane of the Table. And because the Plane KMN, cuts the Table
and

and this Planè NMXZ, therefore the common Sections shall be parallel.

But these common Sections are MN and DE. Therefore MN and DE are parallel; but MN is parallel to AB (by the *Hypothesis*) therefore DE is also parallel to AB.
Q. E. D.

C O R O L. I.

Therefore if MN and HL are two Lines parallel to the Ground Line, their Perspectives DE and PR, shall be parallel to one another in the Table.

C O R O L. II.

If any Line NM parallel to the ground Line, be bisected in O, a Ray drawn from the Eye to the Point of Bisection, shall bisect the Perspective of the said Line DE, in C,

For the Triangles KCE and KOM are Similar. Therefore $OM : CE :: KO : KC$.

Again, the Triangles KON and KCD are Similar.

Therefore $ON : DC :: KO : KC$.

Therefore $OM : CE :: ON : DC$.

But $OM = ON$, therefore $CE = DC$.

C O R O L.

C O R O L L. III.

The same things being supposed as before, I say the Line NM is to its Perspective, as the Distance between the Foot of the Eye's Perpendicular and the Line NM, to the Distance of the Eye from the Table.

For the $\triangle^{les} KDE$ and KNM are Similar, therefore $NM : DE :: KO : KC$:

Again, the $\triangle^{les} KGO$ and CSO are Similar;

Therefore $KO : KC :: GO : GS$.

Therefore $NM : DE :: GO : GS$, but $KF = GS$,

Therefore $NM : DE :: GO : KF$. Q. E. D.

P R O P. XIII.

The Perspectives of all Lines perpendicular to the ground Plane, will, if produced in the Table, be perpendicular to the ground Line.

C O N S T R U C T. (Fig. 11.)

Let CD be a Perpendicular to the ground Plane; and let the erect Plane $RCSDT$, passing thro' the Line CD , be parallel to the Table. From the Points C, D ,

C, D, draw AC, AD, to the Eye at A.
And let the Triangular Plane of Rayes,
ACD, make EM for its common Section
with the Plane of the Table.

DEMONSTRATION.

Because CD and EM, are the common
Sections of *two* parallel Planes by a *third*
Plane, they shall be parallel to one an-
other : Therefore EM if produced, shall
be perpendicular to the ground Line HLP.
The same may be demonstrated of NO
the Perspective of IK. Therefore, &c.
Q. E. D.

COROLLARY I.

Hence the Perspectives of all Perpendi-
culars to the ground Plane, are parallel
one to another in the Table.

COROLLARY II.

The Perpendicular CD, is to its Per-
spective EM, as the Sum of the Distances
of the Eye, and of that Perpendicular from
the Table, to the Distance of the Eye
from the Table ; or, as the Distance be-
tween the Foot of the Eyes Perpendicular,
and the Perpendicular seen, to the Di-
stance of the Eye from the Table.

For

For from Similar \triangle^{ls} ACD and AEM,
'tis $DC : ME :: AD : AM$.

And from Similar \triangle^{ls} ADF and MDL,
it is $AD : AM :: FD : FL$.

Therefore $DC : ME :: FD : FL$.
Q. E. D.

COROL. III.

If two or more Perpendiculars to the
Ground Plane, which are of *equal Height*,
do also stand at equal Distances from the
Table; their Perspectives shall be equal.

Let the Perpendicular $DC =$ Perpendi-
cular IK ; and the Distance $LD = IH$.

And let ME be the Perspective of CD ,
and NO the Perspective of KI . It was
shewn in the Proposition, that $CD : ME ::$
 $FD :: LD$, therefore for the same Reason,
 $IK : NO :: GI : HI$. But $HI = LD$ (by
the Hypothesis) and since $FL = AB =$
 GH , therefore $GI = FD$, therefore $CD :$
 $ME :: IK : NO$; but $CD = IK$, therefore
 $EM = NO$. *Q. E. D.*

COROL. IV.

Any Perpendicular to the ground Plane, is
to its Perspective; as a Parallel to the ground
Line, at the same Distance from the Ta-
ble, is to its Perspective. Because they are
on both Sides proportional to the Lines

FD

FD and FL; as appears by comparing Cor. II. of this Proposition, with Cor. III. of Proposition XII.

And therefore, if the Perpendicular and the Parallel, are equal in the Length, their Perspectives will be equal also.

C O R O L. V.

The *Distance* of the *Object* and *Eye* from the *Table* continuing; the *Perspectives* of the *same* *Perpendicular*, are equal, whether the *Eye* be plac'd at a *less* or a *greater* Height.

The *Object* PG (Fig. 12) the *Table* TK, the *Eye* at the two different Elevations A and S, in the same *Perpendicular* AC. Draw the *Rayes* AP, AG, SP, SG, intersecting the *Table* in the Points q, o, p, e.

Now, PG : qo :: AG : Ao (Similar \triangle^ls , APG, Aqo) :: CG : CK (because AC parallel TK).

Again, PG : pe :: SG : Se (Similar \triangle^ls , SPG, Spe) :: CG : CK (because of the same Parallels.)

Therefore PG : qo :: PG : pe, therefore qo = pe. Q. E. D.

C O R O L.

C. O R O L. VI.

The *Height* of the Eye continuing, as also the *Distance* between the Object and Table: The Perspectives of the same Perpendicular, to the Eye at *two several Distances*, are in the Ratio compounded, of the *direct* Ratio of the Distances of the Eye from the Table, and the *Reciprocal*, of the Distances between the Object and the Eye.

The Object PG, as before, its Perspectives to the Eye at A and D, qo, and gb, respectively; determin'd by the Intersections of the Rayes AP, AG, DP, DG, with the Table TK.

'Tis, $PG : qo :: AG : AO$ (Similar \triangle^ls , APG, Aqo)

$AG : AO :: CG : CK$,

Therefore $PG : qo :: CG : CK$.

Again, $PG : gb :: DG : Db$ (Similar \triangle^ls , DPG, Dbg)

$DG : Db :: EG : EK$,

Therefore $PG : gb :: EG : EK$.

$\therefore qo : gb :: \frac{PG \times CK}{CG} : \frac{PG \times EK}{EG} :$

$\frac{CK}{CG} : \frac{EK}{EG} :: CK \times EG : EK \times CG$.

Q. E. D.

From

From hence it follows, that the Perspective, to the *remoter* Eye, is greater than that to the *nearer*. For since $EG = EK + KG$, and $CG = CK + KG$, therefore $qo : gb :: CK \times EK + CK \times KG : EK \times CK + EK \times KG$; but $EK > CK$, therefore $EK \times KG > CK \times KG$, $\therefore EK \times KG + EK \times CK > CK \times KG + EK \times CK$, $\therefore gb > qo$. Q. E. D.

S C H O L.

By what has been demonstrated at Prop. II. and (with their respective *Corollaries*) may be determin'd, whatever relates to the Proportion between the *Perspectives* of any *direct* Lines, and of any Lines *Perpendicular to the ground Plane*; whether they be *contiguous* to the ground Line, and to the ground Plane, or *not contiguous*, and that for all the various Cases, of different *Heights and Distances* of the Eye. 'Twould be too tedious a Work, to go through them all here. 'Tis sufficient to have shewn the way, both by *Precepts and Examples*, how to proceed in any of them that may occur. But for a farther Help, *this present Figure* may be considered, wherein the Eye is plac'd at several Elevations and Distances from the Table, and the Perspectives of *both Sorts* of Lines mention'd, are distinctly

F

repre-

represented, to put the Reader, the more easily, into the Way of arguing out any of these Particulars. Thus *eg. gr.* the Perspective of the *direct* Line GH, not contiguous to the Table, is the same with the Perspective of the Line GP, perpendicular to the Ground Plane, the Eye being, in either Case at A. And so of the rest.

P R O P. XIV.

If any direct Line be divided into any Number of equal Parts, the Perspectives of those equal Parts shall be unequal.

CONSTRUCT. (See Fig. 12.)

Let the direct Line be DF, the two equal Parts BG, GF, their Perspectives NL, and LM, determin'd by the Intersection of the Rayes AB, AG, AF, with the Line SD, drawn from the Point of Incidence D, to the Point of Sight S. From the Point N, draw NO, parallel to the Line DF, and from the Point C, draw RC, parallel to AF.

DEMONSTRATION.

The Δ^{ts} ABG and ANC; AGH, ACO are Similar, therefore $AG:AC::BG:NC$. Also

Also for the same Reason, $AG : AC :: GF : CO$, therefore $BG : GF :: NC : CO$, but $BG = GF$, therefore $NC = CO$. Again, the $\triangle^{\text{ls}} NMO$, NRC are Similar, therefore $NC : CO :: NR : RM$, therefore $NR = RM$.

In the $\triangle^{\text{l}} AGB$, the $\angle^{\text{l}} AGB$ is $>$ than the $\angle^{\text{l}} AFB$, but $MON = AFB$, and $LCN = AGB$, therefore $LCN > MON$. Now RC , parallel MO , therefore $RCN = MON$, therefore $LCN > RCN$, therefore $LN > RN$. But $RN = RM$, therefore $LN > RM$, therefore $NL > LM$.

Q. E. D.

C O R O L.

Hence it follows, that the Divisions of any *Radial* Line, in the Perspective Table, which answer to any *equal* Divisions of a *direct* Line; are not only *unequal*: But also that the Parts grow less and less, as they approach nearer in that *Radial*, to the *Point of Sight*.

P R O P. XV.

If a Line be inclined, by any Angle whatsoever, to a Transverse Line in the ground Plane; its Perspective shall make the same Angle with the Perspective of that

F 2

Trans-

Transverse Line in the Table. (See Fig. 13.)

CONSTRUCTION.

Let the inclin'd Line be AC making any \angle as ACB, with the Transverse Line BC; from any Point as A, in the Line AC, let fall the Perpendicular AB. From the Eye at R, draw the Rayes AR, RC, RB, intersecting the Table in the Points a, b, c , which are therefore the Perspectives of A, B, C.

DEMONSTRATION.

Since BC is parallel to the ground Line NS (by Hypoth.) therefore bc parallel NS by Prop. XII. Farther, since AB perpendicular BC, therefore also by Prop. XIII. ab will be perpendicular bc ; so that the \triangle^{ls} ABC, abc are Rectangular at B, and b . Now by Corol. IV. Pro. XIII. it appears that $AB : BC :: ab : bc$, because both are in Proportion of FB : Fn. Therefore the \triangle^{ls} ABC and abc are Similar, having their Sides about the equal Angles proportional. Therefore the \angle^{le} $abc = \angle^{\text{le}}$ ABC. Q. E. D.

PROP.

P R O P. XVI.

If in the ground Plane, there be drawn any Number of Parallels to the ground Line, being all of the same Bigness; and at the Extremities of them be erected Perpendiculars to the ground Plane all of the same Height; the Perspectives of these Parallels and Perpendiculars, shall divide all the Radials, drawn through the Extremities of the said Perspectives, in the same Proportion.

CONSTRUCT. (Fig. 14.)

Let the Parallels to the *ground Line*, be OH, MI, NK, included between the same Parallels EN, FK, and the respective Perpendiculars HR, QI, PK, whose Tops are terminated in the Line RP, parallel to FK. Produce the Line RP till it cuts the Table in *d*; and draw the Radial Cd. Let S, V, W, be the Perspectives of N, M, O, and m, n, o, those of K, I, H, and X; Y, Z those of P, Q, R, Lastly, draw the Radials CSE, CmF, CXd.

DEMONSTRAT.

By Prop. XII. Cor. I. Sm, VN, Wo, are Parallels. Therefore $SV : VW :: mn : no$. Again, by Prop. XIII. Cor. I. Xm, Yn, Zo, are Parallels, therefore $mn : no :: XY : YZ$; therefore $SV : VW :: XY : YZ$. Therefore all the *Radials* are cut proportionally. Q. E. D.

The Practice of PERSPECTIVE.

The Practice of Perspective is Twofold, *Direct* and *Inverse*.

The *direct* Method, is that, by which we trace out the Appearance of any *given Object*, upon the Plane of the Table.

The *Inverse*, that, by which from the *Perspective given*, we go back to the Object it self; and so by a sort of *Linear Investigation*, shew the Work to be rightly done. This is very useful and necessary in some Cases, where a Doubt may arise concerning the Exactness of an Operation; and in any Case, 'tis very pleasant, thus to bring what is done to a regular Examen.

The *Geometricians* have their *Synthesit* and *Analysis*, or *Compositive* and *Resolutive* Methods; and the *Analysts*, their *Direct* and *Inverse* Method of Fluxions. And as
tis

'tis a sure Proof, that a *Fluent* is rightly determin'd, when the Fluxion thereof is exactly equal, to the Fluxion at first propos'd; so 'tis certain likewise, that the Work is right in *Perspective*, when by a fair Process from what is done, we can come back to the true *original* Object it self. And the *Parallel* would be exact in all Respects, if we did but shift Names, and call that the *Inverse*, which before we call'd the *Direct* Part of the Practice of *Perspective*: We shall exemplifie *both* these Branches distinctly. But to proceed.

Hitherto we have represented the *Perspective* Table as standing *sideways*, with Respect to the Eye of the Person that looks on the Page, where the Figures are drawn. And 'tis certain that this serves, to give the clearest Idea, of the Demonstrations, of the Propositions, that are advanced in this Science. But now it will be necessary to represent the Table after another manner; that is *foreright* or *direct* to the Eye of the Reader: So that whereas, before, it was imagin'd to stand at *Right Angles* to the Plane of the Page, now we are to conceive it as *lying in that Plane*; this sort of Representation, being the most commodious for Operation and Practice.

I shall begin with the *first Branch* of the Practice of Perspective, *viz. How to delineate the true Scenographical Appearance of any Object given*; or how to proceed from, the Object to the Perspective; which is the *direct Method*.

And in order to the making all things here as easie as may be, I premise, that the Method of *determining the Perspective of a Point*, being shewn at Prop. X, and the Corollary thereof; the Perspectives of all *Lines and Figures*, are from thence likewise determinable.

The Perspective of a *Right Line*, is had, by finding, and joining the Perspectives, of its *Extremities*.

The Perspective of a *Rectilineal Plane Figure*, is had by determining the Perspectives of all its *Sides*.

The Perspectives of *Crooked Lines*, or *Crooked-lin'd Plane Figures*, are determin'd (at least exactly enough for Practice) by carrying a Crooked Line, thro' the Perspectives of a sufficient Number of Points.

The Perspective of a *Solid*, whether *Rectilineal* or *Curvilineal*, is determin'd by finding first, the Perspective of the *Base*, and then setting off the Perspectives of the *Heights*, from their proper Points of Seat in the Base, and joining the *Extremities*.

To

To go on a little farther, with these *General Directions*; let it be observ'd, that the Perspectives of all Lines, which cut the Ground Line, at *Right Angles* are to be carry'd up, to the *Point of Sight*; or if at oblique Angles, then to some *Accidental Point*, determin'd according to *Carol. VI.* and *Schol. Prop. IX.*

And in General, that the Perspectives of all Lines parallel to each other, do run up into one and the same Point, in the Table; by the aforesaid *Prop. IX.*

That the Perspectives, of all Lines *parallel* to the Section or Ground Line, are to be drawn parallel to it, upon the Table; by *Prop. XII.*

That the Perspectives of all Lines *perpendicular* to the Ground Plane, are to be drawn in the Table, perpendicular to the Ground Line; by *Prop. XIII.*

That Lines *inclined* in the ground Plane, are to be drawn with the same Angle of Inclination, in the Table; by *Prop. XV.*

That the Parts of the Perspective become *unequal*, and *shorten* more and more, the nearer they come towards the *Point of Sight*; by *Prop. XIV.*

These Directions relate more specially and immediately to the Practice; though those which flow from the rest of the *Propositions* and their *Corollaries*, are all of them
such,

such, as will be useful in some Case or other this Way. Thus (for Example) it may be of great Use to an Artist that desires to be exact, to consider what is shewn at *Prop. XI, XII, XIII.* with their Corollaries, about the *Augmentation or Diminution* of the Perspective, upon the various Heights and Distances of the Eye : And to know in what Proportion of Magnitude, the Perspectives of *parallel and perpendicular* Lines, are to be drawn upon the Table, to those Lines themselves.

And therefore, as these Uses will be easily found out and made, by those who shall take the right Course to join good Knowledge in *Geometry*, to this Part of *Opticks* : So I shall insist no farther upon that Matter, but *come to propose some Problems*, such as may serve to exercise the Rules before demonstrated.

And for the more effectual attaining this End, we shall shew how they are to be done : 1. By *the more common and expeditious way of a Point of Sight, and a Point of Distance* ; and how, 2. By the Help of the *Accidental Points*.

PROP.

P R O P. XVII. P R O B. I.
(Fig. 15. N^o. 1.)

To find the Seat of a Point in the Perspective Table.

By a Point of Sight and Distance.

Let the Eye be A, the Point of *Sight* B, the Point of *Distance* D, the Point whose Perspective, or Seat in the Table, is required, F. The Line FD is perpendicular to the ground Line GR, wherein is taken $ED = DF$. $D = C$

Then the *Radial* DB drawn from the Point of *Incidence* D, cuts the Line of *Distance* CE in *f*, which is the Seat of the given Point F, in the Perspective Table.

By the Accidental Points. (Fig. 15. N^o. 2, and 3.)

The *former* of these Figures will shew the Reason and Demonstration of this Way of practising, by the Accidental Points, the Table being represented *side-ways*; and the *latter*, the more ready and expeditious Way of Practice it self, the View being here *foreright*. And in both, the several correspondent Points are mark'd exactly with the same Letters, that the Reader might the better understand the Agreement betwixt them. Let

Let the Eye be at A, its Height AB, the Plane of the Table CDEF, the ground Line EF.

The Difference between these Figures, is, that whereas the Object IKL at N^o. 2. is represented very distinctly as lying in the ground Plane, and is not at all confounded, with its Image in the Table OPQ; at N^o. 3. the Object *ikl* seems to be confounded with its Perspective *opq*, although they are not to be conceived, as both lying in the *same Plane*, but *ikl* out *behind* in the ground Plane, and *opq* in the Plane of the Table, erected perpendicularly upon the Line *ef*; the seeming Coincidence of the Planes, arising from the present Position and View.

Let I be a Point given in the ground Plane, (Fig. 15. N^o. 2, 3.) whose Seat in the Table is to be determin'd.

From the given Point produce any two Lines, at Liberty, to cut the ground Line, as IN, IM.

From B the Foot of the Eye's Perpendicular, draw BF, BH, || to IM, IN, respectively. At the Points F, H, (in the ground Line) erect the Perpendiculars FD, HG, each equal to AB.

Join these Points D, G, with the Points M, N, respectively, and where the Lines DM, GN, intersect, as in O, will be the
Seat

Seat of the Point I in the Table. (This being demonstrated at *Pro. IX*, and its *Corollaries*, I shall not need to offer any thing of the Reason of it here.)

PROP. XVIII. PROB. II. (Fig. 16.)

To find the Scenographick Contraction of a Right Line, drawn in any Position to the ground Line.

By a Point of Sight and Distance.

The Points A, B, C, asbefore. The Line given IH; and from the Points I, H, the Perpendiculars EI, H G, to which DE and FG, in the Ground Line GD, are respectively equal. From the Points of Incidence EG are drawn the *Radials* EC, GC, which are intersected by the Lines of Distance BD, BF, in the Points *i*, *h*, which Points are the Perspectives of I, H, and therefore being joyned with a strait Line, give *ih* for the Perspective of the Line IH, which was sought.

Were the given Line *parallel* to the ground Line, as *ex. gr.* IK; its Perspective is determin'd with less Trouble still. For having found the Seat of the Point I in *i*, we need only draw from thence a Line *parallel* to DG, which cuts the other *Radial* CG in *k*, and so makes *ik* for the Perspective of IK.

There

There is no Need to add any thing about the Perspective of a Line *perpendicular* to the ground Line. The Figure it self, sufficiently shews that Matter; as *ex. gr.* in the Lines EI and GK, whose Perspectives are Ei and GK.

To do the same by the *Accidental* Points.

1. If it be an *Infinite* Right Line, whose Scenographick Contraction we would have, as *ex. gr.* MK, infinitely produced towards K, and cutting the Table in M (Fig. 15. N^o. 2, and 3.) we have nothing to do, but to draw BF \parallel to MK, and having erected FD perpendicular to FE, and \equiv AB, to join the Points D and M: So is DM the Perspective of the infinite Line MK.

2. If a *Finite* Line, as IM, and *contiguous* to the Table; we need only draw any other Line at Liberty, as NI, thro' the Point I the Extremity thereof; and then determine (by the last *Prob.*) O, the Seat of I, in the Table: For then joining the Points M, O, that Line is the Perspective Contraction of IM.

3. If a *Finite* Line, and not contiguous to the Table, as LI; this may be done two several Ways:

1. By

1. By *Two* Accidental Points.

This is done by drawing two Lines at Liberty thro' the Extremities of the given Line; as LK, KI, to cut the Ground Line in R and M, and producing LI it self, to cut the ground Line in N; then having determin'd C, D, the proper *accidental* Points, to which the said Lines run up in the Table, as also G, the Point to which the Line LI, infinitely produced, runs up; draw the Lines CR, DM, GN: For then CR and DM, do cut off from the Line GN, a Segment OP, which is the *Scenographic* Contraction of LI sought.

2. By *One* Accidental Point only.

From the Extremities L, I, draw two Lines *parallel* to each other, till they cut the ground Line; and by what has been shew'd before, find the proper accidental Point, to which those *Parallels* converge on the Table.

Then having (as before) produced LI to the ground Line in N, and found G the proper accidental Point, to which it runs up in the Table. If two Lines are drawn from the Point, to which the aforesaid *Parallels* converge on the Table, to the Points where they cut the ground Line; these will

will cut off from the Line NG, the true Perspective of LI.

P R O B. III.

To represent any given Angle in Perspective. (Fig. 15. N^o. 1, 2, 3.)

This is so plain and easie an Operation, after what has been said about *Points* and *Lines*; that there ought to be but few Words, made about it. *Ex. gr.* Suppose the Angle *kli* (Fig. 15. N^o. 3.) were given. Having produced the containing Sides *lk*, *li*, to the ground Line in *r*, *n*, and carry'd Lines from *b* the Foot of the Eye's perpendicular, parallel to them respectively, viz. *be*, *bh*, and erected the Perpendiculars, *ec*, *hg* = *ab*, and lastly joined the Points *c* and *r*, *g* and *n* with right Lines: I say those Lines *cr*, *gn*, form an Angle *cgp*, or *npr*, equal in *Scenographick Representation*, to the Angle *kli*.

P R O B. IV. (Fig. 18.)

To find the Perspective of a Line, Perpendicular, to the ground Plane.

This Problem is absolutely necessary, in order to the setting any sort of *Solid* in Perspective;

Perspective ; and therefore ought to be diligently explain'd.

Let the Perpendicular given be NM, its *Seat* in the ground Plane I, its Distance from the Table IB, the Height OB=NM, set off in the Table from the Point of Incidence B. The Lines OA, BA, *Radials* carried from the Points O, B. In the Radial BA, the Point C, is the perspective of I, the Point of *Seat*. From C is drawn CD, parallel to OB, terminated in D by the Radial OA. The Line CD is the Perspective of NM.

Or thus: From any Point in the ground Line, as E, set off the Perpendicular EF=BO=NM, and having determin'd the Point C (as before) draw CG parallel to the ground Line, to cut EA in G, then will GH perpendicular to the Ground Line, terminated by the Line FA, be the Perspective sought. Or (which is the same thing) GH and CD will be equal to one another.

For AB : AC :: OB : DC, Similar \triangle^ls AOB, DOC, and FE : HG :: AE : AG, Similar \triangle^ls , AFE, AHG, and AE : AG :: AB : AC, Similar \triangle^ls ABE, ACG,
 \therefore FE : HG :: AB : AC,
 \therefore FE : HG :: OB : DC,
 But FE=OB, \therefore HG=DC.

G

And

And it may be much more convenient sometimes, thus to find the Perspective of an upright Line, *apart from the rest of the Perspective*, and afterwards transfer it to its own proper Place in the Perspective, then to set it off there at first.

For when the Case is such, that a great many Perpendiculars are to be carry'd up from the Perspectives of their several Points of Seat in the Figure ; by the Multiplicity of Lines, the whole will be apt to be rendered confused.

If it were required to determine the Perspective, of a Line, any way *inclin'd* to the ground Plane ; it's readily done thus.

Let fall a Perpendicular from the Top of the *inclin'd* Line, to the ground Plane. Find the Perspective of that Perpendicular ; as also the Perspective Seat of the Foot of the *inclin'd* Line.

Draw a Line from the Top of the aforesaid Perspective Height, to the Seat of the Foot of the given *inclin'd* Line, which will be the Perspective sought.

PROP. XXI. PROB. V. (Fig. 19.)

To find the Perspective of a Triangle, in any Position to the Table.

**By a Point of Sight and Distance.*

Let KLM be the Triangle propos'd. KE, LH, IM, are Perpendiculars from the Angles, to the ground Line; and from the Points E, H, I, are drawn the *Radials* EB, HB and IB. The Perpendicular's KE, &c. laid off in the Ground Line, give the Points D, F, G, the Lines drawn to which from C, intersecting the Radials in *k, l, m*, determine the Perspective of the Triangle KLM.

By the Accidental Points.

Let the Triangle be KIL (Fig. 15. N^o. 2, 3.) produce the Sides, to cut the ground Line in R, N, M, and then from B, drawing Parallels to them, in E, H, F, erect the Perpendiculars EC, HG, FD, and join the Points C, G, D, with R, N, M, respectively: So have we by the Intersections of these Lines, the Triangle OPQ in the Table, for the Perspective of IKL.

After this Instance, I shall not need to illustrate the Method of drawing Pieces of Perspective, by the Help of the *Accidental* Points, in any other Figures whatsoever; unless perhaps where 'tis nearer and more expeditious, to work that way than the other.

PROP. XXII. PROB. VI. (Fig. 20.)

To represent in Perspective, a Square, divided into several little Squares; one Side being parallel to the ground Line.

Let the Square be AK GD. Let AY, ZH, In, qK, Perpendiculars to the ground Line, in which the Distances being laid off, mark out the Points Q, R, S, T, V, W, X. Let the Lines YO, ZO, nO, qO, be *Radials*. The Lines PQ, PR, &c. carry'd from the Point of *Distance* C, intersect the Radial Oq, in g, m, l, k, from whence drawing dg, me, lb, ka, parallel to QR, we have the desir'd Divisions in Perspective.

PROP. XXIII. PROB. VII. (Fig. 21.)

To do the same when one Angle of the Square is turned to the Table.

This is most conveniently done, by the Help of the *Accidental* Points.

The

The Lines of *Incidence* being drawn, and the Distances laid off in the Ground Line (as usual) let $nSWZ$ be the perspective outlines of the Square $MFNI$. The Accidental Points are E and C , the Lines AE and AC , being parallel to MF and FI , by *Cor. X. Prop. IX.* Let the Points W, X, Y, Z , be the Perspectives of I, H, G, F . Then laying a Ruler thro' them and the Point E , mark out the Points s, q, r, n , which joyned with W, X, Y, Z , will divide the Sides nS, ZW . So likewise, the Points a, o, p, Z , being found, a Ruler laid over them and the Point C , will divide the other two Sides of the Perspective, nz, SW . And the Intersections of these cross Lines, will determine the Perspective of the little Squares, in the *Original*.

P R O P. XXIV. P R O B. VIII.

To set any Rectilineal Plane Figure, whatsoever, whether Regular, or Irregular, in Perspective.

Find by the Rules afore-given, the Seats of the several *Angular* Points of the Polygon given, in the Perspective Table. These Points joyn'd with Right Lines, will give the Perspective of the Figure propos'd.

PROP. XXV. PROB. IX. (Fig. 22.)

To set any upright Prism, or Pyramid in Perspective.

For a Prism.

Let the Base be ABCDE, whose Side AB is parallel to the Ground Line, and the Height NO. By the foregoing Rules, find the Perspective of the Base, which let be FGHIK, having drawn a Line from O, any Point in the ground Line, to L the Point of Sight; erect the Perpendicular NO equal to the Height of the Prism; and join NL. At the Points F, G, H, I, K, a Ruler being laid parallel to the ground Line OZ. Intersects the Line LO, in the Points g, h, i, from whence drawing gR, hQ, iP parallel to NO; these shall be (by Prop. IV.) the perspective Heights of the Prism at those several Points. Where note, that there are but three different Heights in all, for those that are to be raised upon K and I, will be equal one to another, so likewise with those at F and H, tho' less than the former. Lastly, that at G, will be the least of all.

The Reason of this is, because the Side the Pentagon AB is supposed to be parallel to the ground Line. So that now laying down the perspective Plane or Base FGHIK,

in

in a Place apart by it self; upon KI erect the Perpendiculars KP, IT, equal to the Perpendicular iP, and on the Points F, H, erect FQ, HS equal to h Q; and Lastly, from G erect GR=gR, and join the Points PQRST, so you have the Perspective of the whole Solid.

For a *Pyramid*.

Let the Base, as before, be ABCDE, the Height NO, and W the Center of the Base.

Having drawn the *Perspective Plan* FGHIK, and therein determin'd *w* the Seat of W; we have nothing to do, but from thence to carry a || to the Ground Line, and at its Intersection with the Radial LO, to take off (as before) the proper *Perspective Altitude*, which is afterwards to be erected upon the Point *w* in the Plan. For Lines drawn from the Extremity hereof, to Angles F, G, H, I, K, will compleat the Perspective of the Pyramid.

P R O P. XXVI. • P R O B. X.

To set any Sort of Oblique Prism or Pyramid, in Perspective,

For *Pyramids*; we shall need only one Perpendicular let fall from the *Vortex*, to the Ground Plane.

G 4

Having

Having therefore drawn the perspective Plan, and determin'd whereabouts in the Table, the Seat of that Point in the Ground Plane, on which the Perpendicular from the *Vertex* falls, will be; as also having determin'd (by the Rules above given) the just Measure, of the Perspective, of the *said Perpendicular*: Lastly, having set off this Perspective Altitude, from its proper Seat in the *Plan*: There is no more to be done, but to draw Lines, from the Extremity thereof, to the several Angles of the aforesaid Perspective Base or Plan.

In *Prisms*, the Matter is a little more troublesome, because of the *many* Perpendiculars required to be let fall on the ground Plane, from the *upper* Angles of the Body.

(As *eg. gr.* in the oblique Quadrangular Prism CDFEGHIK (Fig. 22.) from whose upper Angles, are let fall the Perpendiculars CL, FN, DM, EO; and whose Side GH, and consequently IK, for facilitating the Work, I suppose to be parallel to the Ground Line OQ.)

However, those Perpendiculars being let fall, and the *Measures of their Perspectives*, *pm, qo*, duly determin'd, as also their
Points

Points of Seat m, b, n, o , in the Table : Then if those *Perspective Altitudes* qo, pm , be each set off, in its *proper Measure*, from its *proper Point of Seat* in the Table, *viz.* pm from b and m , and qo from n and o ; and Lines, *viz.* cg, dh, fi, ek , drawn from the Extremities of them, *viz.* c, d, f, e , to the *correspondent* Angles of the *Perspective Plan*, or *Base*, *viz.* g, h, i, k ; and *Lastly*, if those Extremities themselves be *aply joined* with *Right Lines*, *viz.* cd, de, ef, fc ; the *Perspective* of the oblique Prism proposed, will be compleated, *viz.* $ikghfed$.

S C H O L.

There is in these Cases, Choice to be made of some such Position of a Body to the Table, *that the Work may be the easiest and shortest possible.*

Thus for Example, the foremention'd Prism CDEFGHIK, (at Fig. 22.) was placed with its Side GH parallel to the Ground Line OQ ; and consequently the two entire Surfaces of it, GHCD, IKFE, parallel to the Plane of the Table. By this Means the two Perpendiculars CL, DM, being at equal Distances from the Table, are represented in *Perspective* by one and the same Line pm . And so the other

other two FN, EO, by the Line *go*. So that we have but these two Perspective Heights, to determine in this Case. Whereas, had one of the *Angles* as G, been turn'd towards the Table, we might have had three or four several Perspective Altitudes, to have determin'd. If the *Diagonal* IH or NM, had been parallel to QO; then there had been *Three*, of which, that for CL would have been biggest; those for FN and DM, less than the former, tho' equal to one another, and that for OE least of all, as being the farthest from the Table.

But if NM were not parallel to the Table, it is plain, there must have been *Four* several Perspective Heights found; since the four Perpendiculars above-mention'd, would in that Case have stood, at four several unequal Distances from the Table.

The like is to be observed, in other Figures.

PROP. XXVII. PROB. XI.

To set any Solid, contained under Plane Surfaces (whether Regular or Irregular) in Perspective.

The Operation for the *oblique Prism*, (in the last Problem) will be a sufficient Direction for this, without a particular Figure

gure. *Ex. gr.* Let the Body propos'd be an *Icosaedron*, which we will suppose set on its Base, which is one of the *Twenty Equilateral* Triangles, under which it is contain'd. This Body having twelve solid Angles; when it is set on one of its containing Triangles as a Base; there are nine of the said Angles, remaining above the *ground Plane*, from each of which, Perpendiculars are to be let fall. And here now we shall find, the Use of what was hinted at the *Scholium* of the last Problem; about the Choice of such a Position, that the Work may be the shortest possible. For if the Equilateral Triangle, which is the Base, be turned with one of its Sides \parallel to the Table; we shall have the Perspectives of but *six* Perpendiculars to determine. And the same also, if one Angle of the Base be directed to the Table in such sort; that a Perpendicular let fall from thence to the opposite Base of the Equilateral Triangle, would, if produc'd, cut the ground Line at right Angles. For this is the same Case as the former. But if it be set in any other Position, we shall have *nine* several Perpendiculars to set in Perspective.

Having therefore let fall Perpendiculars from the elevated Angles to the ground Plane, and set the Base (whereon the Body stands) in Perspective; and *Lastly* determin'd

termin'd the *proper Heights* of those several Perpendiculars, upon the Perspective Table, and set them off from their proper *Points of Seat* therein : If then the Points are aptly joyn'd (as the Inspection of such a Solid will best direct) the *Perspective out-Lines* of the Body will be compleated.

And thus may any Body whatsoever, contain'd under Rectilineal Surfaces, be expeditiously set in *Perspective*.

PROP. XXVIII. PROB. XII.

To set all Sorts of Cones and Cylinders, in Perspective.

The Rule will proceed here in like manner as at *Prob. X*, for *Pyramids* and *Prisms* ; abating only the Difference arising from the *Bases*, which here are *Curvilineal Figures*, (*viz. Circles*) and there, *Rectilineal ones*.

At *Cor. I. Prop. V*. I have shewn how to determine, when a Circle, shall come an exact Circle, into the Perspective Table : That is, when the Perspective of a Circle shall be a Circle. And it must always be either a *Circle* or an *Ellipsis*, when the Table stands as we now suppose it, *viz. Perpendicular to the Ground Plane*.

For

For that the Perspective of a Circular Line, may upon other Suppositions, be any other of the Conical Sections, as well as an Ellipsis; we have shewn already at Prop. VI.

So that therefore if *such Distance and Height of the Eye*, be made Choice of, that the Base of the given Cone or Cylinder, be a Circle in the Perspective Table; if the *Seat* of the Center, and the perspective Magnitude of the Radius, be likewise determin'd, (by Prob. I. and II.) the Base is describ'd with little Trouble.

But if any other Position be chosen, so that the circular Base, of the Cone or Cylinder, comes into an *Ellipsis* upon the Table; it may be described sufficiently well for common Practice; by dividing the Circumference of the Circle into a good Number of Parts, and having found the Perspectives of the several Points of the Division, *to carry a Crooked Line thro' them*, with a steady Hand. Or to go more *Geometrically* to work; the Ellipsis may be describ'd, by finding the Longer and Shorter Axes of it upon the Table (as shall be shewn by and by) or by many other Ways besides.

SCHOL.

S C H O L. I.

One thing is to be minded here with Regard to *Cylinders* (for there is no farther Difficulty at all in *Cones*) and that is, that tho' such a Position be made Choice of, that the *Lower Base*, *ex. gr.* should be a Circle in the Table, yet the *Upper Base* cannot at the same time, be so too, but will be an Ellipsis: Or *vice versa*, if the Upper be a Circle, the *Lower* will be an Ellipsis.

The Reason of which is most evident, from that aforesaid *Cor. I. Prop. VI.*

For since the *Distance* from the Table, being *given*, there is a *particular Altitude* of the Eye required, in order to make the Perspective of a Circle, to be a Circle; and since in an *Upright Cylinder*, the Upper and Lower Bases are both equally distant from the Table, but the Eye has not an equal Elevation over them both: 'Tis plain, that if the *Height* of the Eye over the *Lower Base* in the ground Plane, be so proportion'd to its Distance from the Table, that the Perspective thereof shall be a Circle; the *Less* Height of the Eye over the *Upper Base*, cannot be proportion'd to the same Distance from the Table, so as to produce the same Effect.

So

So that in an upright Cylinder, the Perspective Appearances, of the upper and lower Basis, can never be of the same Kind, but if one be a *Circle*, the other will be an *Ellipsis*; that is, supposing the Cylinder it self, and the *Eye*, to retain the same Position, and Distance from the Table.

S C H O L. II.

To find the longer and shorter Axes, of this Ellipsis, upon the Table.

Imagine two Diameters in the Circular Base of the Cylinder, cutting each other at Right Angles, so that one of them be parallel to the Table, and the other consequently perpendicular thereto.

The Perspectives of these two Diameters, found by Prop. XI, and XII. will be the Axes of the Ellipsis to be described upon the Table.

Now the Length of the Diameter being given, the Scenographick Contractions thereof, in these two Positions, are easily found; by knowing the Distances it lies at from the Table, *viz.* the single Distance of the Diameter which is *parallel*, and the Distances of the two Extremities, of that, which is *perpendicular*, to the Table.

These

These, I say, being given, the Perspectives are found, by the Rules aforementioned; and consequently, the Axes determin'd.

P R O P. XXIX. P R O B. XIII.

(Fig. 23, 24, 25.)

To set a Row of Bodies in Perspective.

We will take a Series of *Parallelipipids*, and suppose them rang'd in such Order, on one side the Eye, that their Sides which are perpendicular to the Table, may lie all in a right Line. And we will imagine one of them to be *contiguous to the Table*; which will in some Measure shorten the Work.

Let (Fig. 23.) the Point of Sight be at A, the Point of Distance B, Z the Square Base, and WQ one of the Including Rectangles of the *Parallelipipid* propos'd.

Make $CD = WX$, $DH = XQ$, as also $HW = DH$. Draw the Lines HA, DA, CA, from the Points H, D, C, to the Point of Sight A.

Joyn the Points BW, with a right Line, intersecting the Radial AH in G, from whence a parallel to the Ground Line DWZ, cuts the Radial AD in F, and determines the Trapezium HDGF, for the Perspective Base of the first *Parallelipipid*.

Lastly,

Lastly, from F raise a Perpendicular to DZ, (or which is all one, a Parallel to DC, which we suppose perpendicular to DZ) which intersects the Radial AC in E; and thus we have the diminish'd Altitude EF, and consequently DCEF for the Perspective Representation, of the Side - Rectangle or Surface of that first Parallelipiped.

And from this Base and Side - Superficies, all the rest that finishes the Perspective of the said Body, is determin'd.

And thus are the Bases and Sides found for the rest, *viz.* OMNI, LMIK, for the Second,; VSQT, RSPQ, for the Third, and so on.

Note, If the first Parallelipiped had not been supposed contiguous to the Table, the Side DH, could not have lien in the Ground Line, but would have been at some Distance from it, and so would have been *diminished*, and not appear'd in its full Bigness, as now it does.

And the Reason, why we made HW = HD, is because the Base Z being supposed a Square, that side thereof which lies opposite to DH, cannot be distant from the Table any more or less, than the Length of DH it self; for (as I said) this first Parallelipiped is *contiguous* to the Table.

Nor is there any other Difference in the Work, when one Body is placed *contigu-*

ous to the Table, and when they are all at a Distance from it, than only this, that in the former Case 'tis shorter, by as much, as finding the Perspective of one Line amounts to.

The Perspective Plans and Elevations being found, as at Fig. 23. the Perspectives of the *whole Solids* are set together very easily, as at Fig. 24. *Ex. gr.* The Plan GFHD, Fig. 23. being transferred to *gfhd*, Fig. 24. upon the Points *h, d*, erect the Perpendiculars *dc, ah*, = DC, and at *g, f*, the Perpendiculars *gb, ef*, = EF, and joyning the Points at top and bottom, with right Lines, (as in the Fig.) the out Lines of the Parallelipiped are completed. In like Manner for the Second and Third, transfer the Plans OMNI, VSTQ, into *omni, vstq*, and erect the Perpendiculars KI, LM, in the Second, and PQ, RS, in the Third, each in its proper Place; and so fill up the out Lines for them, and the rest, if there were more.

Lastly, The Parallelipipids compleatly finish'd and shaded, appear as at C, D, E, Fig. 25.

PROP.

PROP. XXX. PROB. XII.
(Fig. 26.)

To represent a Pedestal, in Perspective.

This is done very easily, by Help of the Directions given at the last Problem.

Let the *Geometrick Plan* or *Base* be F, the *Geometrick Elevation* or *Profile* C, the *Point of Sight* at A, and of *Distance* B; the Lines CK, and HK perpendicular to each other.

Having put the Base F into Perspective, as at E, and drawn the occult Lines *b, b, b*, &c. from the several Angles of the Elevation C, perpendicular to HK, as also the occult Lines *c, c*, &c. parallel to CK: Suppose the *Perspective Elevation* D, to be completed.

This being set in two opposite Sides of the perspective Plan, as was done for the *Parallelepipids* (at Fig. 24.) will complete the Perspective outlines of the Pedestal.

And the whole adorn'd with its proper *Shades*, will appear as at G.

P R O P. XXXII. P R O B. XII.
(Fig. 27.)

To delineate the Perspective Representation of the Roof, Pavement and Side-walls, of any long Room or Entry, whose Dimensions are given.

Suppose BC the Height, CD the Breadth, CH the Length, of the *Place* propos'd, IE the *Eye's* Height, FE its Distance; all taken off in their proper Measures, from the *same Scale* of equal Parts. The *Table* is imagin'd to stand perpendicular upon CD, (which is therefore our *Ground Line*) and the Spectatours Position such, that a Perpendicular from his Foot to CD, falls thereon at the Point F; which Perpendicular is EF.

Having drawn the *Rectangle* BCAD, one of whose Sides is the Height, and the other the Breadth (and is the *Geometrick* Section of the Room by a *vertical* Plane, Breadth-ways) produce the Line EF, till it cuts BA in K. In the Line FK, set off the Eyes Height IE from F to N, and draw the Lines NC, ND.

By *Corol. I. Prop. XI.* find the *Perspective* Contraction, of the *Length* of the *Place*, viz. GH; saying,

As

As the Sum of the *Length*, and the *Eyes*
Distance from the Table,
 Is to the *Height* of the Eye;
 So is the *Length* it self,
 To its Perspective *Contraction*, or Foreshort-
 ning.

That is, $GH + FE : IE :: GH :$ to a Fourth,
 or the *Foreshortning* sought.

This being taken off, from the *Scale*
 used before, for the Geometrick Delinea-
 tion, is to be laid in the Line FN, from
 F *ex. gr.* to L. Thro' the Point L, draw
 PO parallel CD, intersecting the Lines
 NC, ND, in the Points R, S.

Again, By *Corol. II. Prop. XIII.* find
 the Perspective *Contraction*, of the Rooms
 Height BC; saying,

As the Sum of the *Length*, and the *Eyes*
Distance,
 Is to the *Eyes Distance*,
 So is the *Height* it self (of the Place)
 To its Perspective *Contraction*.

That is, $GH + FE : FE :: BC :$ to a
 Fourth, which is the *Contraction* sought.

Let this be taken off from the same
Scale, and laid in the Line FK, from the
 Point L (determin'd before) to W *ex. gr.*

or which is the same thing, set it off in the Lines RQ, ST, which are parallel BC, from the Points R, or S, to Y, or X. Which done, compleat the Rectangle RSYX, and draw the Lines BY, AX. Or else, having set off LW, thro' W draw NM parallel to BA, which cuts the Lines NB, NA, in the Points Y, X, and so does the very same thing. And thus all the outlines, of the intended Peice of Perspective, are drawn.

For the Trapezium CRSD, is the Representation of the *Floor*, BYXA of the *Roof*, BYRC and AXSD of the *Side Walls*.

PROP. XXXII. PROB. XIV. (Fig. 28.)

To represent an Arch in Perspective.

This Work is so like that of the foregoing Problem, that there need not be much said of it.

The Eye's *Height* is set off from N to L. The Line MN is the Perspective Contraction of the *Length* (or *Depth*) and GM, of the *Height*; answering to LE, and WL, in the *last Figure*, and obtained by the same Proportions. The Figure HGKPV is here in this Case, what YXRS was in that; the *Circular Arch*, being carry'd thro'

thro' the three Points H, G, K, which are determin'd by the above-mention'd Rules, which give the foreshortning of the Length and Heights.

The Lines BH, *nm*, EK, *rq*, OW, TV, YZ, PR, are directed to the *Point of Sight* L; as in the former Case, CR, DS, BY, AX, were carry'd towards N. In a Word; TVPR is the Perspective Representation of the *Ground Area*, BHTV and FKPR of the *Sides*, and HBAEK, of the *Concave Superficies* of the Arch.

And by the Help of these two Examples, may any other Delineations of the like Nature be perform'd.

PROP. XXXIII. PROB. XV, (Fig. 29.)

To perform the Practice of Perspective, without Regard to Point of Distance, or any Accidental Point whatsoever.

Let the Table be ABDI, the Point of Sight C, the Eyes Height CE, any visible Point in the Ground Plane, P, whose Incidence on the *Ground Line*, is at H, and its Distance PH.

. CONSTRUCTION.

Draw the Line DO in any Angle at Liberty to DI. Make DN *eq. gr.* or DL,
H 4

if it happened to be less) equal to the Eye's Distance from the Table, NO (or LM) = PH, the Distance of the given Point from the Ground Line. Draw OI (or MI) and NK (or LK) parallel thereto. Thro' K draw KS parallel to BI, and lastly CH intersecting KS in Q.

I say that Q is the proper Place or Seat of the Point P, in the Perspective Table.

DEMONSTRATION.

Call the Perspective of the Line PH, π .
By Prop. XI. $PH : \pi :: DN + PH : CH$,

$$\therefore DN + PH : PH :: CH : \pi,$$

$$\therefore DN : PH :: CH - \pi : \pi,$$

But $NO = PH$ (Construct.)

$$\therefore DN : NO :: CH - \pi : \pi.$$

Again, $DN : NO :: DK : KI :: CQ : QH$
(Similar Δ^{ls} .)

$$\therefore CQ : QH :: CH - \pi : \pi,$$

$$\therefore QH : CH :: \pi : CH - \pi + \pi = CH,$$

$$\therefore QH = \pi.$$

Therefore Q is the Seat of the given Point P, in the Perspective Table. Q. E. D.

SCHOL.

S C H O L

'Tis sufficient, to have shewn the way of tracing out by this Method, the Seat of any Point in the Perspective Table; since from hence any Figure whatsoever may be easily laid down.

But the Ways for doing these things are endless; and therefore I shall leave it to every one, to invent or follow what Method he pleases.

Having now dispatched what Problems are necessary, to render any Studious Person sufficiently well acquainted with the Practice of the *Direct* Method of Perspective, upon *Upright* or *Vertical* Tables.

I shall add one or two Propositions, tending to the farther Illustration, and Improvement, of this curious Subject; and then come to shew how we are to proceed upon *Horizontal* and *Inclin'd* Tables.

After that, in a few Instances, I shall exemplifie the *Inverse* Method of Perspective; that is, how to go back from the *Perspective*, to the *Original*, or Object, whose Perspective it is. And the foregoing Rules being well understood, there will be no Need, to say much upon that Matter.

P R O B.

P R O P. XXXIV. THEOR.
(Fig. 7.)

Every Deformation, is a regular Piece of Perspective, upon the very same Plane; to the Eye, plac'd at another Height and Distance.

I have already said something in general at *Prop. VII.*, and its *Coroll.* concerning the Nature of *Optical Deformation*, and its Distinction from what we commonly call *Perspective*. It is shewn there, as also at *Schol. Prop. VI.* that this is no other than an *inverted* Sort of Perspective; and that upon the Account of the *different Order*, in which the Object and the Table lie, with respect to the Eye.

But I shall now demonstrate other Reasons for its being so, and shew how these Practices, do all fall within the Rules of the ordinary Perspective.

CONSTRUCTION.

Let the Eye be at K, its Height KV, the Ground Plane (which serves as a Table in this Case) VSTDE; on which is perpendicularly erected the Plane Figure ABPN, which is projected by the Eye at K, into PNDE.

We

We will suppose the Figure ABPN to be a Rectangle, and consequently, its Deformation E NDE, is a Trapezium; whose Side PN is parallel DE.

Upon DE erect a Plane, as MQFH, perpendicular to the Ground Plane, which produce out both Ways at Liberty. Let fall a Perpendicular thereto, from the Eye at K; which cuts it in the Point G.

DEMONSTRATION.

The Lines PD, NE, concur in V, the Foot of the Eye's Perpendicular (by *Construction*, at *Prop. VII.*) And since the Plane MQFH is (by the Hypothesis) perpendicular to the Plane STDE; therefore if the Former be made a *Ground Plane*, and the Latter a *Perspective Table*; its evident that the Parallels MF, QH infinitely produced, will run up to some Point of *Sight*, in the aforesaid Plane STDE. Now if KG be made the Height of the Eye, and KV its Distance from the Table; then the Point of *Sight* is V, and the Lines DV, EV, the Perspectives of the Parallels DF, EH, infinitely produced; and therefore PD, NE, are the Perspectives of some Finite Portions of those Parallels. Therefore the *Eye* being at K, the Ground Plane MQFH, the *Table* VSTDE, the *Eye's Height* KG, its

its *Distance* KV; the Trapezium PDNE, is the true *Perspective*, of some Portion of the Rectangle DEFH produced. But the same Trapezium was the Deformation of the Rectangle ABPN, to the same Eye at K, its Height being KV, its Distance VO, and the Table STDE. Therefore, &c.
Q. E. D.

S C H O L I.

We may easily determine, what Part of the Infinite Rectangle, FDEH, the Deformation PNDE, is the common *Perspective* of, upon the Table VSTDE.

For drawing AI or BI, parallel to VD or VE, and VO perpendicular to PN, and produced to cut DE in T; from Similar Δ^s , arises KI ($=KV-IV=KV-BN$) : IB ($=VN$) :: KV : VE.

Again, VN : PN :: VE : DE; and Lastly, VE—NV=NE.

Now then, if we take DE for the *ground Line*, and erect the Infinite Plane VSTDE perpendicularly thereon, as our *new Table*, and also at Right Angles there to the Infinite Plane MQFH, for our *ground Plane*; we have then the *Height*, and *Distance* of the Eye, as also the *Perspective* Contraction of some Portion of a *Direct Line* to find (by Prop. XI.) the Length of that *Direct Line* it self. That is, we have KG ($=VT$) and KV,

- KV, and NE, to find the *Length*, of which NE is the Scenographick Contraction.

And this being found, we may pronounce, *viz.* that a Rectangle one of whose Sides is DE, and the other the Line thus found, being put into Perspective by the commo Rules, for the *Height* and *Distance* of the Eye, GK, and KV; will be the true *Deformation* of the given Rectangle AN, upon the same Plane, but with the Height and Distance of the Eye, KV, and VO.

S C H O L. II.

Since *Shadows* are nothing but the *Deformations*, or *Projections* of the *Out-lines*, of Bodies, upon certain Planes; and since we have demonstrated, how the Practice of Deformations is reducible to that of the *Common Perspective*: 'Tis plain, that the Practice of *Sciagraphy*, or of determining the Shadows of Bodies, is likewise reducible to the same; so that from the necessary Data (of the *Figure* of the Body, and) of the *Height* and *Distance* of the *Light*, we may settle the proper Height and Distance of the *Eye*, that the Shadow may be drawn upon a *Table*, as an ordinary Piece of Perspective.

L E M.

L E M. (Fig. 30.)

If there be any Number of Planes, cutting each other in the same Right Line; and another Plane be drawn Perpendicular to their common Section: Then, the common Sections of the former Planes with this last Plane, shall be all at right Angles, to the common Section of those said Planes.

The Planes MLGQ, MKFP, MIEO, whose Ground Lines, LQ, KP, IO, are supposed Parallel; cut each other in the Line MBN; and are all of them cut, by the Plane ACD, in the Lines BE, BF, BG, respectively. The Line MB, is supposed perpendicular to the Plane ACD, at the Point B. These things supposed; I say that MB, the common Section of the afore-said Planes with one another, shall be at right Angles to the Lines BE, BF, BG, the common Sections of the same Planes, with the Plane ACD.

This is so manifest from *Eucl. Elem. 11.* that there is no need of insisting on the Proof of it.

C O R O L.

The Triangles MBR, NBR, MBT, NBT, &c. lying in the Planes MLBGQ, MKBFP, &c. are all of them Rectangular at B. PROP.

PROP. XXXV. THEOR.
(Fig. 30.)

If an Object in the Ground Plane, appear to the Eye, in Direct Vision, in any Points whatsoever of an Upright Perspective Table; then, if a Plane Speculum, were substituted instead of the Table, and the Eye placed at the same Distance, on the other Side thereof; it would receive the true Perspective of the Object, by this reflex'd Vision, as before by the Direct.

DEMONSTRATION.

Let the Eye be at M, the Table ACDH, any Object as OPQ, in the Ground Plane; whose Perspective, or Image in *Direct* Vision, is STR.

The common Sections, of the Planes MLGQ, MKFP, MIEO, with the Plane of the Table, *viz.* GR, FT, ES, do all run up to the Point B; as has been demonstrated at Prop. IX.

And (by the foregoing *Lemma*) these Lines RB, TB, SB, are each of them perpendicular to MB, the common Section of all the Planes.

Let us suppose then in the next Place, that the Table ACDH were a *Plane Speculum*,

lum, and that the Visual Rayes PT, QR, OS, were reflected thereby, into the Lines Tq, Sr, Rp, at the Points T, R, S; at which Points we imagine Tc, Rb, Sa, to be erected perpendicularly, to the Plane of the Speculum; and consequently to lie in the Planes MKBFP, MLBGQ, MIBEO. And let the Line MB, which is by the Supposition perpendicular to the Plane of the Glass, be produced out in the other Side at Liberty, as BV. By the known Laws of *Catoptricks*, the Angle $PTc = cTq$, $QRb = bRp$, $OSa = aSr$. But because Rb *ex. gr.* is perpendicular to the Plane of the Speculum, therefore the Angles bRB and bRG, are right ones. And consequently, the Angle $BRp =$ the Angle QRG. But $QRG = MRB$; therefore $MRB = BRp$.

Let Rp cut the Line MV, in N.

Then, since $MBR = NBR$ (being right ones, by *Corol.* to *Lem.* foregoing) and $MRB = NRB$, and BR common to both Triangles; 'tis plain that $MB = BN$.

In like Manner, we will demonstrate; that the Angle $BTM = BTN$, and $BSM = BSN$.

And consequently, that the other *reflex'd* visual Rayes, Tq, Sr, do also meet in the same Point N.

And

And therefore were the Eye placed at N, it would see the Object OPQ, by the Means of the Glass; appearing at *opq*, on the other Side, just as OPQ it self appears, on this Side.

That is; as the Eye at M, sees the Object OPQ, in *Direct* Vision, appearing on the Table, as SRT; so the Eye at N (equally removed) sees the same Object, by the Help of the *Glass*, appearing at *opq*, just as far *behind* the Glass; as OPQ is *before* it, and in the very same Form too, *viz.* That of SRT, which is the same Perspective. Q. E. D.

C O R O L.

Hence *Plane Looking-Glasses*, may be usefully apply'd, to the Purpose of drawing Pieces of Perspective.

P R O P. XXXVL T H E O R.

(Fig. 31.)

Images, formed by Reflexion from Plane Glasses, are regular Pieces of Perspective; in which the Height, and Distance of the Eye, as also the proper Point of Sight, are all easily determinable.

This Proposition, differs much from that which went before, For what was shew'd
I there,

there, was this; that the Eye, by the Help of a Plane Glass, might have the very same true Perspective of an Object, which it would have, for any given Height and Distance of the Eye, in *Direct* Vision.

But what is to be proved here, is this; that a Light being plac'd before a Plane Glass, the Image of the same Glass, formed by the reflex'd Light; *ex. gr.* upon the Roof or *Cieling* of a Room, will be a regular Piece of Perspective, whose Point of Sight is somewhere determinable, upon the aforesaid Roof or *Cieling*. So that the Looking-Glass, is here, not only the *Instrument* to reflect, but also the *Object* it self, whose Form is reflected.

For as in other Cases; a Speculum receiving the Species of some ordinary Object, reverberates it, and makes that Object visible to the Eye at a proper Distance and Position; so here, a Speculum receiving the Rayes of an *actual Light*, or Luminous Body; returns it own Form or Shape, upon a neighbouring Plane; which will be very different, according to the Positions of the *Glass*, the *Plane*, and the *Luminary* it self.

DEMON.

D E M O N.

Let ABCD be a plane Looking-Glass, *ex. gr.* of a Rectangular Form, the Light at E, which falling on the Glass in the Rayes EC, EA, EB, ED, is reflected up to the Ceiling TNOL; and figures there, the Speculum it rebounded from, in the Form of a Trapezium *abcd*, whose two Sides, *ex. gr.* *ab*, and *cd*, are parallel to each other.

Suppose the Plane of the Glass, if continued, to cut the Ground-floor in the Line ST, which is cross'd at right Angles in I, with the Line EF, at one End of which, stands the Light E.

Take IF (*behind* the Glass) = IE, the Distance of the Light *before* it; and from the Point F, erect a Perpendicular to the Floor, *viz.* FG, which strikes the Ceiling in G.

I say G is the proper *Point of Sight*, for the Perspective *cabd*; or 'tis that in which the converging Sides of the Trapezium, *ca*, *db*, if produced, would meet.

By *Prop. XXXIV. Theor.* If the Eye be plac'd at F, and were supposed to project the Object ABCD upon the Ceiling TNOL (which we suppose parallel to the Horizon) the *Deformation abcd* will be a regular

Piece of Perspective, upon the very same Plane; in which the Point of Sight will be G, and the Eyes *Distance* from the Table, FG, and its *Altitude*, a Perpendicular from F to a *vertical* Plane passing thro' $\kappa\delta$.

And *vice versâ*, the Perspective *appears* upon the Ceiling, appears to the Eye at F, as the Rectangle ABCD, upon the Vertical Plane ABST.

But by *Prop. XXXV. Theor.* the Speculum being ABCD, if instead of the *Light*, an Eye were placed at E; it would receive the same Appearance, of the Object, *appears*, by this *reflex'd* Vision, at E, which it had before in *direct* Vision, when plac'd on the other Side at F; the Distance IF being = IE.

That is, the Rayes $F\alpha$, $F\beta$, $F\kappa$, $F\delta$, would be reflected into EA, EB, EC, ED. And therefore, on the other Hand, if instead of the Eye, a Light be placed at E, the Incident Rayes EA, EB, &c, will be reflected by the Glass ABCD, into $A\alpha$, $B\beta$, $C\kappa$, $D\delta$, which if produced would all meet in F.

So that 'tis plain, the Projection *appears*, and the reflected Image *abcd*, perfectly coincide with one another. And therefore the said reflected Image *abcd*, is a regular Piece of Perspective, whose Point of Sight is G. Q. E. D.

COROL

C O R O L.

Hence again, Plane Glasses may be applied to Perspective Uses ; but after a manner very different, from what was suggested at *Corol.* of the foregoing *Prop.*

Of Horizontal Perspective.

Tho' we have hitherto been professedly considering only *Upright* Tables, and how to trace the Appearances of Objects on them ; yet the *last Proposition* intimates so much of the Reason of the Practice on *Horizontal Tables* likewise, that we have not only a very easie and natural Transition from thence, to this Speculation, but shall also find it necessary to say less of that Matter than otherwise, upon the Score of what we have there demonstrated.

• P R O P. XXXVII. T H E O R.
(Fig. 32.)

'Tis the same thing to draw a Piece of Perspective, upon an Horizontal Table ; as upon a vertical Table, the Eye's Height and Distance being alternately charged,

I 3

• DEMON,

DEMONSTRATION.

Let the Eye be at A; GO, the Ground Plane, AG its Height above the same; BE, an Horizontal Plane above the Eye, DH a Plane perpendicular to the two former, AB, the Eye's Distance from the *Horizontal* Plane, AN, its Distance from the *Vertical* Plane, D any visible Point in the Plane DH.

From the Eye at A draw the visual Ray AD, cutting the *Horizontal* Table in C.

'Tis plain that C, is the Perspective of the Point D (lying in the vertical Ground Plane ED) in the *Horizontal* Table BE, to the Eye, at A, whose Distance from the Table is AB, and from the vertical Ground Plane, is $AN = BE$.

And therefore since the Angle AND is a right one; if, while the Eye continues still in the same Point at A, we suppose HD, which was before a *vertical* Ground Plane, now to become an *Horizontal* one; as also AN and BE, which before were Horizontal, now to be set perpendicular to the Horizon: It is evident, that by this Change of Position, all things are now reduc'd to the common Case of *Upright* Tables.

For

For DH is the Ground Plane, BE the Vertical or Upright Table, AN the Height of the Eye above the Ground Plane, AB its Distance from the Table, and B the Point of Sight thereon.

And in either Case, the Point C, the Perspective of D, continues in exactly the same Place and Position in the Table. And for the same Reason, would the Perspectives of any other Points in the Ground Plane HD, be the same when BE stands vertical, as when it lies parallel to the *Horizon*.

And therefore, 'tis the same thing to draw a piece of Perspective upon, &c. Q. E. D.

C O R O L. I.

The Rule therefore for Practice, is this, *viz.* To draw upon the *Horizontal* Table BE, after the manner that we would do, if it were an *Upright* one; wherein B should be the *Point of Sight*, AN the *Eye's Height*, and AB its *Distance* from the Table.

C O R O L. II.

The same Rules hold, whether Pieces of *Perspective* of this Kind, are to be drawn on Planes above or below the *Eye*; as *ex. gr.* whether on the *Roof* or *Ceiling* of a Church

Church, so as to be view'd from the Floor, or on the *Pavement*, so as to be view'd from a Gallery.

C O R O L. III.

Were a *Pedestal* or *Column* (or a Rank of each) standing perpendicular to the Horizon, to be represented in this sort of Perspective, *ex. gr.* upon the *Cieling*; it would be the same thing, as to place the same *Pedestal* or *Column*, parallel to the Horizon in the Ground Plane; and then draw the Perspective of it, upon an *Upright Table*, *ex. gr.* a *Wall*.

For thus; if we suppose, the Line DE (for Example) to be a Pillar, perpendicular to the Horizon GO; it will be all one, to represent this in Perspective upon the Cieling BE; as it would be, if DH being the Horizon or Ground Plane, and consequently the Pillar DE lying flat thereon: we should draw the Perspective of it, upon the *Wall*, or vertical Table BE.

And it is to be observed; that in either Case, the *Circles* keep their proper Form in the Perspective; as lying in a Position parallel to the Table, and consequently (by *Prop. V.*) being *Circles* there likewise.

And the *Sides* of the Columns, are *Direct Lines*, or such as are perpendicular to the Table,

Table, and therefore in the Table are carried up to the Point of *Sight* B.

And upon this Account *Horizontal* Perspective is indeed much easier, than *Vertical*, or that which is perform'd upon an Upright Table; contrary to what the *Painters* generally imagine. For 'tis plain, that 'tis easier, *ex. gr.* to put a Column, that lies flat on the Ground Plane, into Perspective, upon an Upright Table; then 'tis to draw the Perspective of that same Column, standing perpendicular to the Ground Plane, upon the same vertical Table. For the Difference lies here; That in the *former* Case, the *Circles* (as I said) keep their Form in the Table; and the *Sides* likewise, are all carry'd up to the Point of *Sight*; whereas in the *latter*, the *Sides* are to be *shortned* upon the Table, and also the *Circles* cannot keep their Form; for the Reason of which, I refer to *Schol. I. Prop. 5.* But now; we have shewn, that 'tis the same thing to represent an *Upright* Column, in Perspective, upon an *Horizontal* Table; that 'tis to represent that same Column, lying flat in the Ground Plane, upon an *Upright* Table. And therefore, I say the Practice of *Horizontal* Perspective, is in many Respects much easier than that of *Vertical*.

SCHOL.

S C H O L.

We may easily determine where an *Upright* Table ought to be plac'd, that the same Object, may have the same Perspective thereon, which it has on any *Horizontal* Table; the Eye keeping the same Position in each Case.

As if (Fig. 32.) the Eye being at A, we were to find where a *vertical* Table ought to stand, as in *ex. gr.* where in HN or SP, &c. that so the Perspective of the Line DE thereon may be the same as that of the said Line DE, upon the *Horizontal* Table BE; the Eye continuing still at A.

Put $HO = DE$. Then by Similar Angles $HO : HI :: GO : AG$, also $DE : CE :: DN : AN$; wherefore the Perspectives CE and

HI, are as $\frac{DE \times AN}{DN}$, and $\frac{HO \times AG}{GO}$, or

$\frac{AN}{DN}$ and $\frac{AG}{GO}$. Therefore if $CE = HI$, then

will $AN : DN :: AG : GO$, and so the Rectangular Triangles ADN, AGO are Similar, and therefore the Angle ADN, = AOG, or $DAB = OAN$, or $DAB + DAN = OAN + DAN$; but $DAB + DAN$, or BAN , is a Right Angle by *Construction*, and therefore $OAN + DAN$ must be so too. And consequently we must draw

AQ,

AQ perpendicular to AD, and having set off $QP=DE$, erect the Perpendicular PS; for then on this Table, shall the Perspective PR, be equal to CE, upon the Horizontal Table; the Eye in both Cases being at A. *Q. E. I.*

C O R O L .

If the Figures BN and AN, were Squares; then in this Case the Vertical Table ought to stand in NH, in order to our having the Perspective $NH=CE$.

For they being Squares, then $AG=AN=EN=GH$; and because $HO=DE$ by Supposition, $\therefore GO=DN$, and since CE

$HI :: \frac{DE \times AN}{DN} : \frac{HO \times AG}{GO}$, 'tis plain the

Scenographick Projections, on these two Tables are equal to each other.

Of the Practice of Perspective, on Tables Inclined to the Horizon.

Though the Rules of this Perspective, have much Affinity, with those before demonstrated for *Vertical* or *Upright* Tables; yet there is not that Sort of Coincidence, or Agreement betwixt them, that some of the Writers of this Science have imagin'd.

Thus

Thus (for Example) M. Lamy's Account of this Matter, as we find it in his *Perspective*, printed at London 1702; is far from being either clear or genuine: and that abating all Mistakes of the Press.

Let (Fig. 33.) BG be the Ground Plane, SE an *Upright* Table; HE an *Inclin'd* one, the Eye's Height, AB, its Distance from the *Vertical* Table, AP, the Line BC = AB, and \parallel^1 EH, and from C, a Line as CN \parallel^1 BG, striking the *Inclin'd* Table in the Point N, and therefore = BE.

Suppose the Eye at C, viewing any Point as G in the Ground Plane, and by the Visual Ray CG, making its Perspective Seat, in the *Inclin'd* Table at F; so that EF, is the Perspective of FG, on the said Table, to the Eye at C.

Now he tells us, that (keeping the same Point of Station E) if we bring the Table EH into the *Upright* Position ES; and the *Spectatour* moves back from BC to BA; that then the Point C coinciding with A, and N with P, the Point F which is the Perspective of G, to the Eye at C, upon the *Inclin'd* Table, will also coincide with D, which is the Perspective of the same Point G, to the Eye at A, upon the *Upright* Table: that is, that EF = ED.

And indeed it is true, that if the Postures of the *Spectatour* and the Table, are thus shifted

shifted as he supposes; the Points C and A, N and P, will coincide. Also I allow that F and D will do so too; or that $EF = ED$.

For from Sim. \triangle^{ls} $BG : AB :: EG : ED$,
Also, $BG : BC :: EG : EF$,

Therefore $ED = EF$; tho' at the same time, his way of proving it, is (to say no more) very confus'd and odd.

However, that we pass by; and grant him, that ED is equal to EF . And what if it be so; what follows from thence? Why then says he; *The Perspective of G, after this Change of the Positions, will be found in the very same Point of the Picture*; that is, when the Table is set upright, and the Spectatour has erected himself likewise; it will be just were it was, when both were inclin'd. Very well! And now then, what is the Rule arising from hence, in order to Practice? Why he tells us, *That we are to draw the Perspective of an Object upon the upright Table ES, according to the Rules before given; making AB (=BC=EN) the Height of the Eye, and its Distance, AP (=BE=CN;) the Point of Sight P, in this Table, being the same with N, in the other; because the Lines EN and EP are equal. And when this is done, we are only to set the Table ES, back again into the Place EH, and the Spectatour*

Spectatour to betake himself to his stooping Posture, so as to place his Eye in the Point C; and then the Perspective will answer Expectation.

But this Gentleman to be sure, did not consider, that tho' the Perspective of G, on at the *Inclin'd Table*, with respect to the Eye C; does thus coincide with the Perspectives of the same Point, on the *upright Table*, to the Eye at A; and tho' there will be (by Vertue of the same Demonstration) the like Coincidence, as to the Perspectives of any other Points, taken in *the same Line EG*; yet when he comes to take a Point, that lies in some other Line; and not in EG; *he must then of necessity shift his Eye from C, into some other Place, in order to obtain this Coincidence of Perspectives, upon the two Tables.* And this will be demonstrably evident to any one, if the Tables, which are here represented by *strait Lines* only, be but represented in their proper Dimensions as *Plane Figures*. And therefore, as many different Lines of Incidence as there are, in which the Points in the *Ground Plane*, whose Perspectives he would find, are posited: *So many several Removes and Shiftings, of the Eye from C, must there necessarily be:* that is, the Spectatour must put himself, into the same Variety of new Places and Postures; in order to have the Perspectives, of the
the

the Points of an *Object*, on an *Inclin'd Table*, coincident with the Perspectives, of the same Points on an *upright Table*. And what an easie and practicable Method this would be, of drawing a piece of Perspective on an *Inclin'd Table*; I leave it to the World to Judge.

But, which is the main thing of all; he has quite drop'd the true Problem, and substituted an other in the Room of it.

All that is of Use, and which is what a Man would enquire after and expect in the Solution of such a Problem; is *how to draw upon an Inclin'd Table*, ex. gr. *EH*; keeping his perpendicular Posture *AB*, and his Eye, continually in the self same Point at *A*: And not, how he may shift his Eye from *A*, into an other Posture *C*, and there have his Perspective *F*, coincident with *D*, when the Table and Spectatour are both set upright again; and so to go on at this Rate, into an Infinity of Postures and Positions; which in Practice, is to do nothing at all, and therefore to prescribe it, is to teach nothing at all. Certainly, as there are Rules for Drawing upon *Vertical* and *Horizontal Tables*, not incumber'd with any such precarious Changes and Removes of the Eye; so the like Rules be demonstrated for Tables *inclin'd to the Horizon*; and the Principles on both Sides are so near

near a-kin, that the Application is not difficult, to be made from the one to the other.

P R O P. XXXVIII. T H E O R.
(Fig. 34.)

The Point, to which the Perspectives of any Parallels in the Ground Plane, converge upon Tables Inclined to the Horizon; is (as in those which are Vertical) determin'd by the Intersection of the Table, by a Ray passing from the Eye, parallel to the aforesaid Parallels in the Ground Plane.

Let the Eye be at A, its Height AB, the Ground Plane CHDI, the Inclined Table CEFD, any Parallels in the Ground Plane, CLH, DKI; the Line LK parallel to the Ground Line CD. Draw the Visual Plane ALK, whose Section by the Plane of the Inclined Table, is MN; which is therefore the Perspective Representation of the Line KL, upon the said Table.

It's evident, that when the Visual Plane ALK, becomes parallel to the Horizon, the two Sides AL, AK, coincide with each other, and the whole Plane falls into the Right Line AP, which is parallel to the Horizon, and strikes the Table in G, which is the *Point of Sight*.

Hither

Hither tis, that the Lines CM and DN, the Perspectives of CL and DK, do converge; so that CG and DG, are the Perspectives of the Parallels CH, DI, *infinitely produc'd*. All which is most easily demonstrated by the *Prism*, after the manner that we, proceeded at *Prop. IX.* for *upright Tables.* Q. E. D.

P R O P. XXXIX. T H E O R.

The Perspectives of all Lines parallel to the Ground Line, are parallel one to another; upon Inclined Tables as well as Upright ones.

Thus if the Line LK, be parallel to the Ground Line CD, we will demonstrate (as at *Prop. XI.*) that its Perspective MN, shall also be parallel to CD; and therefore, all parallel to one another, Q. E. D.

C O R O L.

Therefore all these Lines, are drawn upon *Inclined Tables*, after the very same Manner as upon those that are *Vertical*.

P R O P. XL. T H E O R.

The Perspectives of Lines, Perpendicular to the Ground Plane, are to be drawn upon Inclined Tables, after a very different Manner, from what they are on upright ones.

For by Prop. XII. upon *Vertical* Tables, these Perspectives, are all *Perpendicular* to the *Ground Line*, and consequently parallel to one another.

The Reason of which is, because the *Visual Planes*, which are all perpendicular to the Horizon, being cut by the Plane of the *Table*, which is likewise perpendicular to the Horizon; their common Sections (*viz.* The Perspectives of the Lines, perpendicular to the Ground Plane) must necessarily, be all of them, perpendicular to the *Ground Line* of the *Table*.

But it can't be thus, when the *Table* is plac'd stooping or inclin'd to the *Ground Plane*. For the *Visual Planes*, which are all perpendicular to the Horizon, being cut by the *Inclined Oblique Plane* of the *Table*; will not make the common Sections or Perspectives, perpendicular to the *Ground Line*; but *inclining*, and that in various Angles of Obliquity.

Nor

Nor is there any more, than one Section only, of the *Inclin'd Table*, by a visual Plane; wherein the *common Section*, is at right Angles to the *Ground Line*.

But one there is; which is, when the *Visual Plane*, cuts the *Inclin'd Table* at Right Angles. In this Case, the Perspective, of a Line perpendicular to the *Ground Plane*, will also be perpendicular to the *Ground Line* of the Table; otherwise not, as any Man may easily satisfy himself, from the common Principles of Geometry.
Q. E. D.

PROP. XLI. THEOR. (Fig. 35.)

If *ADBOp* be the Plane of a Table, inclin'd to the Horizontal Plane *DrOn*; the Eye at *C*, its Height *Cr*, from whence a Perpendicular as *rOn*, is let fall to the Ground Line *DOp*; Lastly, *CB* parallel to *rn*, or the Horizontal Plane, striking the *Inclin'd Table* in *B*: I say, that *n* being any visible Point in the Ground Plane; if we set off *Op = On*, and *AB = CB*, and carrying up the Line *OB* from *O* to *B*, join the Points *A* and *p*, with the right Line *Ap*, intersecting *OB* in *c*; that then the Point *c* shall be the true Perspective Seat, of the given Point *n*, in the Table.

CONSTRUCTION.

From the Eye at C, draw $Cs \parallel^1 Bo$, as also $AD \parallel^1$ to the same Bo , from the Point A. Draw the visual Ray Cn , cutting the Table in the Point a .

DEMONSTRATION.

The Point a , is the *natural* Perspective of the Point n , to the Eye at C, upon the Inclined Table; and by Prop. XXXVIII. this Point a is found some where in the *Radial* Bo . Now I'll demonstrate that a is coincident with c , determin'd by the Intersection of the Lines Ap and Bo , drawn after the Manner, as is express'd in the Proposition.

The Triangles sCn , oan , are Similar;

Therefore, $sn : on :: sC : oa$.

Again, the Triangles ADp , oap , are Similar;

Therefore, $Dp : op :: AD : oa$;

But $sn = so + on = CB + on$ (because of \parallel^s) $= AB + on$ (Construct.) $= Do + oa$ (because of \parallel^s) $= Do + op$ (Construct.) therefore $sn = pD$.

Again; $on = op$ (Construct.)

And $sC = Bo$ (because of Parallels) $= AD$ (because of Parallels) therefore the *Fourth* Terms

Terms of the Proportions are respectively equal, *viz.* $oa=oc$.

Therefore the Points *a* and *c*, do coincide with each other on the Table.

That is, the Point *c*, determin'd by the Intersection of the Lines *Ap* and *Bo*, is coincident with *a*, the *natural* Perspective of *n*, in the Ground Plane.

Therefore the Perspective of the Point *n*, in the Ground Plane, is rightly determined, by laying off in the *Ground Line*, $op=on$, and in the *Horizontal Line*, $BA=BC$, and then drawing *Ap*, to cut the *Radial Bo* in the Point *c*. Q. E. D.

S C H O L.

This Demonstration for Inclined Tables, proceeds exactly after the same manner, with that general one given at *Prop. IX*. For Tables *perpendicular* to the Horizon. All the Difference is; that the Line *CB*, is there *perpendicular* to the Table, and here *oblique*; which necessarily arises from the different Position of the Table in that Case and this: But in both Cases it's parallel to the Horizon, and where it strikes the Table, (as here at *B*) determines the *Point of Sight*, if the Lines *on*, &c. are at Right Angles to the Ground Line *Dp*; or otherwise, some *Accidental Point*.

And those that will take the Pains to draw the Figure out, may accommodate the Demonstration to any Case ; let the Line *on*, in the Ground Plane, lie (as it does here) passing through the Foot of the Eye's Perpendicular *Cr* ; or any other ways on either Side of the Eye. But indeed, the bare Inspection of the Fig. referr'd to at the fore-mention'd Prop. will be a sufficient Proof of the Universality of this Demonstration, for *Inclin'd* Tables, without any more ado..

C O R O L L A R Y I.

Hence then we have a Method of tracing out practically, upon any *Inclin'd* Table, the Perspective Seats, of any given Point or Points, in the Ground Plane ; and consequently of delineating the entire *Scenographic* Appearance, of any Object, upon such a Table.

C O R O L. II.

The Distance BA in the Horizontal Line (which determines A, the *Succedaneous Point of Distance*) being $=CB=os=sr+ro$; is therefore $=$ the perpendicular Distance of the Spectator from the Ground Line, added to the Cotangent of the Tables Inclination ;

tion, the Eye's Height being the Radius. For the Angle $Csr = Bon$, the Tables Horizontal Inclination.

Also oB , the Perpendicular Distance, of the *Point of Sight*, above the Ground Line Dp ; is the Co-Secant of the same Angle: Which Remarks may be useful in Practice.

P R O P. XLII. P R O B. XVI.

To draw upon an Inclined Table, the Perspective of any Line, perpendicular to the Ground Plane.

How all Sorts of Lines *lying* in the Ground Plane, are to be drawn in Perspective upon these Tables; we have shewn already: But how those which are *raised above* the Ground Plane, are to be represented, is a thing of more Difficulty. I shall shew therefore, how we may easily and practically trace out upon a Table, any how inclin'd to the Horizon, the Perspective, of any Line, which stands *erect on the Ground Plane*: it being easie from thence, to draw the Perspectives of any Lines that are *oblique* thereto; as we have intimated before; in *vertical* Perspective.

From the Foot of the *Perpendicular*, where it stands in the *Ground Plane*, carry a Line of *Incidence* to the *Ground Line* of the Table.

On this Line of Incidence, imagine a Plane to be erected perpendicular to the *Ground Plane*; by which Means, it will also be perpendicular to the *Inclin'd Table*; and its *common Section therewith*, will be at Right Angles to the *Ground Line*. This common Section, for Distinction sake, I call *the Perpendicular of the Table*, and is represented in Fig. 35. by the Line Bo. Thro' the Apex or Top of the *Perpendicular*, conceive another Plane to be carried, *parallel* to the *Ground Plane*, whose common Section with the Table, will be a *new Ground Line*, and parallel to the former below.

Where the common Section of this *Horizontal Plane*, and the former *Erect one*, cuts the Table, will be the *new Point of Incidence*, for the Apex of the *Perpendicular*; whose *Distance* also, from the *new Ground Line*, is that Part of the aforesaid common Section of the two Planes, which lies between the *new Point of Incidence*, and the Apex of the *Perpendicular*.

And thus, having the two *Points of Incidence*, viz. for the Foot, and Apex of the *Perpendicular*; as also the *Distance* of each

each from its respective Ground Line; we are only to determine (by the Help of the Rule demonstrated in the last Prop.) the *Perspective Seat* of each of these Points) and so join them with a Right Line; which will be the true *Scenographick Appearance, of the Perpendicular proposed*, upon the Inclining Table.

Now the *Inclination* of the Table, the *Height* of the propos'd Perpendicular, and the *Distance* of its *Foot*, from the Ground Line, being all actually given; it's easie to find, the *Distance* of the *Apex*, from the *new Ground Line*, and whereabouts the *new Point of Incidence* falls in the Table.

For; *Radius*, to *Co-secant* of the Tables Inclination, so the given *Height of the Perpendicular*, to a Fourth; which is equal to the Segment of *the Perpendicular of the Table*, intercepted between the Points of Incidence, of the *Foot* and *Apex* of the Height propos'd.

And as *Radius*, to *Cotangent* of the Tables Inclination, so the given *Height*, to a 4th; which subtracted from the *Distance* of the *Foot* from the Ground Line; gives the *Distance* of the *Apex* from the new Ground Line.

And after this, I believe there cannot be much Difficulty remaining, with Respect to the Practice of Perspective upon these sort of Tables.

The

The Inverse Method of Perspective.

Hitherto we have been conversant, about that Part of the Practice of Perspective, which is very properly call'd the *Direct*; since it is the Method of proceeding, from the Object it self, to its Perspective Appearance; so that knowing the true Form and Position of the *Former*, we can immediately trace out the *Latter* on the Table.

The *Inverse* of this, shews how, by a retrograde Sort of Process, from the Perspective given; to determine the Figure and Situation of the *Original* or Prototype: Which Method I shall now exemplifie in some few Problems, but sufficient to lead the Reader (who is well instructed in the foregoing Practice) into all the Parts and Steps of this.

P R O P. XLIII. P R O B. XVII.

Any Point in the Perspective Table, being given; let it be required, to find its Original Seat in the Ground Plane. (See Fig. 15. N^o. 2, 3.

Let O be a Point given in the Table; thro' which draw any two Lines at Liberty, which produce, till they cut the *Horizontal Line*, *ex. gr.* in G, and D,
and

and the *ground Line* in N and M; so that the entire Lines themselves, are DOM, and GON. From the Points D, G, let fall the Perpendiculars DF, GH, to cut the Ground Line in the Points F, H; and from B, draw the Lines BF, BH. Then from the Points M, N, draw the Lines MK, NL, *parallel* to BF, BH, respectively; which intersect each other in I: I say, that I is the true *Original*, of the Point O in the Perspective Table.

Otherwise thus. (Fig. 15. N°. 1.)

Let *f* be any Point in the Table, the Seat of whose *Original*, in the *Ground Plane*, is required.

Draw a Line from B the *Point of Sight*, thro' the given Point *f*, till it cut the Ground Line in D; at D, erect DF perpendicular to the Ground Line; and having drawn a Line from the *Point of Distance* C, thro' *f*; produce it till it cuts the Ground Line, in E. In the Perpendicular DF, Set off $DF = DE$. Then will F be the Point sought.

The Demonstration of these Practices will be very easie, to those that understand the Reason, of the Operations at Prob. I.

However, for the sake of those, that may desire to see them demonstrated, I shall add the Demonstrations, and that of each Practice distinctly.

DEMON-

DEMON. *of the First Practice.*

The Lines OG and OD, are drawn from the Point O, till they cut the *Horizontal Line* in G and D. And (by *Construction*) the Lines GH and DF, are Perpendiculars from the Points G and D, to the *Ground Line EF*.

Now by *Corol. VI, VII. Prop. IX.* all Lines in the *Ground Plane*, parallel to BF, run up in the *Perspective Table*, to the Point D; as also these which are parallel to BH, converge to G.

But (by *Construct.*) the Lines NL and MK, are respectively parallel to BH and BF.

Therefore the Lines NL and MK in the *Ground Plane*, infinitely produc'd; are the *Originals* answering to the *Perspectives* NG and MD.

Therefore the Point I, which is the Intersection, of the said infinitely produc'd Lines NL and MK, is also the true *Original*, of the Point O in the *Perspective Table*. Q. E. D.

DEMON. *of the Second Practice.*

By *Construct.* the Line DF is perpendicular to the *ground Line*, therefore by
Corol.

Corol. VI. Prop. IX. the Line DB, is the Perspective of DF infinitely produc'd ; or *vice versâ*, DF infinitely produc'd, the Original of BfD; and consequently the Original of the Point *f*, must of Necessity be somewhere in the said Line DF produc'd. But also since C is by *Hypoth.* the Point of Distance, and the Points C, *f*, E, do by *Construct.* lie all in a Right Line ; and moreover since by *Construct.* DF is taken equal to DE ; therefore by *Prop. X.* F is the true Original of the Point *f* in the Table. Q. E. D.

P R O P. XLIV. P R O B. XVIII.

Giving any Line in the Table, to determine its Original, in the Ground Plane.

1. Let the Line given in the Table, be terminated both ways, *viz.* by the Horizontal and the Ground Line ; *ex. gr.* MD.

From the Point D in the Horizontal Line, let fall DF perpendicular to the Ground Line, and intersecting it in F ; then from B, the Foot of the Eye's Perpendicular, draw a Line to F ; and from M (where the Line given cuts the ground Line) draw MK at Liberty, parallel to BF. I say that the Line MK infinitely produc'd, is the true Original or Prototype of MD in the Table. The

The Demonstration is apparent, from what has been said before.

2. Let the Line be terminated by the *ground Line*, and some other Point in the Table; as *ex. gr.* the Line MO.

Having produced the given Line MO, till it cuts the Horizontal Line of the Table in D, and drawn DF, and BF, and MK, in that manner that was shewn just now; we have nothing to do, but only according to *Prob. XVII.* foregoing, to determine the *Original* of the Point O, or what Point in the *ground Plane*, belongs to O in the Table. And having by that Means found the Point I, we have consequently, the Line MI, for the Prototype of MO.

3. Let the Line be terminated by the *Horizontal Line*, and some other Point in the Table; as *ex. gr.* the Line DO.

Produce DO, till it cuts the Ground Line in M, and draw DF, BF, MK, as before; then determining I (in MK) for the *Prototype* of O; all the infinite Production of the Line IK beyond I, will be the true *Original* of DO.

4. Let the Line given in the Table, be terminated neither by the *Horizontal*, nor the *ground Line*; as *ex. gr.* the Line PQ.

Having

Having produc'd it both ways, till it cuts the *Horizontal Line* in C, and the *ground Line* in R; from C, let fall CE perpendicular to the *ground Line*, and draw BE; then from R, extend the Line RL at liberty, parallel to BE; in which Line, the Points K, L, the respective *Originals* of Q, P, may be determin'd (as by *Prob. XVII.*) and consequently KL, for the *Prototype* of the given Line PQ.

Or thus: Having drawn CE, BE, and RL, as before; take any Point in the Table, at Liberty, as O, and from thence carry two Lines at Liberty, thro' the Extremities P and Q, producing them till they cut the *Ground Line* in M, N, and the *Horizontal Line* in D, G; from whence let fall the Perpendiculars DF, GH, and from B, draw the Lines BF, BH. Then from the Points M, N, draw the Lines MK, NL, at Liberty, parallel to BF, BH respectively, which produce till they intersect the Line RL, in the Points KL. I say, the Line KL is the true Original of QP.

PRO P.

PROP. XLV. PROB. XIX.

Any Angle being given in the Table; to find an Angle in the Ground Plane, to which the said Angle in the Table, is equal in Representation.

Suppose the Point P, was taken at Liberty in the Table, through which were drawn the Lines PO, PQ, any how, so that the Angle OPQ be formed; and it be requir'd to determine the Angle in the Ground Plane, to which the said Angle OPQ is equal in Representation.

The Lines containing the given Angle, being produc'd till they cut the *Horizontal Line*, in the Points G, C, and the *ground Line* in N, R; from the Points G, C, let fall the Perpendiculars GH, CE, each equal to the Eye's Perpendicular AB. From B draw the Lines BH, BE, to the Points H and E; and from the Points N and R before determin'd in the *ground Line*, produce the Lines NI and RK, parallel respectively to BH, BE, which intersect each other in the Point L. I say that the Angle RLN, is the Angle sought, *viz.* That to which OPQ in the Table, is equal in Representation.

For

For from the Practices already demonstrated, we will shew, that the *Prototypes* of PO, and PQ, are found in the Lines NI, RK produc'd, and consequently that the Angle of the *Prototypes* NLR, is the true Angle represented by OPQ in the Table.

S C H O L.

After what has been demonstrated of the Practice of the *Inverse* Method of Perspective, with respect to *Lines* and *Angles*; there can be no Difficulty remaining, how to extend the same to *Plane-Figures*, or even to *Solids* themselves. 'Tis true, that will be more laborious; however, there are no new Rules, and 'twill be but a bare Repetition of the Work already done. *Ex. Gr.* If we were to compleat in the *Ground-Plane*, the *Original* of the Plane Figure OPQ in the *Table*.

Having determin'd the Angle NLR, answering to OPQ in the *Table*, find another Angle in the Ground Plane, equal in Representation to some other in the Table, *ex. gr.* to O, or Q.

I'll take O for Example; and producing OO, till it cuts above and below in D and M, and drawing DF and BF as before; I produce MI, parallel to BF, which intersects the Lines RL and NL, before drawn,
L in

in I and K, and so cuts off the Figure IKL, for the true *Prototype* of OQP.

And thus we have gone through all that is of grand use, either in the *Direct* or *Inverse* Practice of Perspective; and I'll venture to say, That one who well understands the foregoing Practices, with their Demonstrations, may successively attempt any Problems whatsoever relating to either of them.

I shall conclude, with some brief Observations, upon a most curious and useful Problem in this Science, which, to the best of my Knowledge, has never been so much as touched upon, by any of those who have written the most *Mathematically* this way; for nothing of this Kind, is ever to be expected from the common Mechanical Practitioners.

The Problem is this, *viz.* Giving an Object in the Ground Plane, with its Distance from the Table, ~~and the Distance of the Eye~~ To find such a proper Distance of the Eye from the Table, that the *Original* or *Prototype* may be to its Perspective on the Table (Area to Area) in any given Ratio of Majority.

Note, I say, in any given Ratio of Majority: For the Section of the *Visual Pyramid* on the Table, will ever be less, than its Basis, in the Ground-Plane. To

To solve this Problem, in one of the most useful Cases in Practice; will be as much ~~as~~ I need do: Those that have a mind to do ~~it~~ in more, may do it at their Leisure.

And though I have actually solv'd some Cases of this Problem ~~Algebraically~~, yet I shall not bring in those Computations here; being resolved to use no other, but pure *Geometrical* Reasonings in this Treatise.

Suppose therefore (at Fig. 6.) that the Object DE, being a Circle in the Ground Plane, it were requir'd that its Perspective upon the Table GD, should be a Circle, and a Circle, whose Area should be to that of DE, in the Ratio of the Line N to the Line M. *where*

The *Height* of the Eye is supposed to be *unknown*, ~~and therefore~~ we will denote by the Line H. *chiefly*

But what is *wanted*, is that particular *Distance* of the Eye from the Table; that the visual Cone may not only be cut *subcontrarily*, by the Plane of the Table; but also that the *Section* may be to the *Basis*, in the assigned Proportion, of N to M. *for the eye's height will easily be found, when this distance is once determined*

Without supposing any thing at all of the Circle, or the *Construction* mention'd before at that Fig 6; we'll imagine the Eye to be at the Point F; being wholly ignorant

whereabout the Point F is, or how far distant from the Table. And therefore imagining the Visual Rayes FE, FD, to be drawn, we'll suppose the Section sought for, to be CD.

Now $\odot DE : \odot DC :: M : N$ (*Hypothesis.*)

That is ; $DE^q : DC^q :: M : N$.

But $DE^q : DC^q :: FE^q : FD^q$ (*Subcontr. Section.*)

And supposing a Perpendicular from the Eye to the *ground Plane*, viz. FB ; a Line = FD, will fall some where on the other Side of it. Let that Line be FA.

Then $DE^q : DC^q :: FE^q : FA^q$.

Because BF (wherever it falls) is perpendicular to the Ground Plane, therefore it is parallel to the Table GP.

Therefore DFB = FDC.

But FDC = FED (*Subcontr. Selt.*)

• Therefore DFB = FED.

But because FA = FD (*Construct.*) and FB is perpendicular (*Construct.*) therefore AFB = DFB, and AB = BD.

Therefore AFB = FED.

Therefore the Angle AFE, must be a Right one.

If

If so, then it must be $FE^q : FA^q :: BE : BA$.

But $FE^q : FA^q :: M : N$ (*Construct.*)

Therefore $BE : BA :: M : N$.

But $BA = BD$;

Therefore $BE : BD :: M : N$.

And, $BE - BD : BD :: M - N : N$;

That is $DE : BD :: M - N : N$.

Therefore BD the proper Distance of the Eye from the Table, is determin'd, *QEF*.

L 3

A N



AN
APPENIDX
CONTAINING
A brief Account of some Things,
of Use, in the ART
OF
PERSPECTIVE.



A N

APPENDIX.

I. Of Scenes for the Stage.

A THEATRE painted according to the Rules of Art, appears a Regular Piece of Perspective; when viewed from a certain Point. Nor are there any other Rules needful to the Understanding, all the *Mathematical* Part of this fine Piece of Theory; than only some of those demonstrated in the foregoing Treatise.

I believe there is none that has written, both so much, and so curiously upon this Subject, as the Famous *Jesuit Andrea Pozzo*, in his two Volumes of *Architecture in Perspective*, especially the *Second*; which therefore, all that are desirous of being informed in these Matters, will do very well to consult. But there is enough in the *First* Volume, to let any Person, into the true *Knowledge, Use and Construction* of

of *Scenes*, who, together with these Rules, actually sees the Disposition of all in a Theatre.

Here the Reader may find, the proper *Dimensions* of the several Parts of a Theatre; the Method of finding the *Point of Sight* therein, and the *Distance* from whence it shall appear a Just Piece of Perspective. Also how the Scenes are dispos'd and directed in their *Grooves*; what their proper Heights are; how by knowing the Width between the *nearest* and *farthest* Grooves, the *Length* of the Theatre, or the Distance of its *Point*, from the Edge of the *Stage*, may be found; with various other curious and useful Practices relating to this Matter.

All which being so amply and particularly treated of, by the aforesaid Author; I shall not need to enlarge on them here, but refer the inquisitive Reader thither, where he will meet with all the Satisfaction he can reasonably desire.

II. *Whether more than one Point of Sight only, be to be admitted in Pieces of Perspective.*

To answer this, 'tis necessary that we distinguish, with the *Excellent Author* just now mention'd.

'Tis

'Tis one thing to speak *in the Gross*, of any Work of large and great Extent ; and another thing, to speak of the *several distinct Parts of that Work*.

In the *former* Sense, more Points of Sight, than one, are to be admitted.

In the *latter*, viz. When we speak of *any particular Part of a Work*, we ought to assign but one Point of Sight only ; and to each several Part, its own proper Point.

Many Points of Sight introduc'd into the same Piece, or where there is one sole entire Design ; would be more prejudicial to the Work, in many Respects, than the making use of one *only*.

For whereas, if one only be made use of, 'tis then plain, that from some *one determin'd Point*, a compleat and *perfect View*, may be taken of the whole Piece ; if several be introduc'd, then there is no one Point, from whence you can have a perfect View of the whole ; but all that can be done, is to view each several Part of it, from its own proper Point.

Besides, good *Painting*, being but an Imitation of Nature ; a Painter is not oblig'd to make his Work appear real, or as the very *Life*, from any Point ; but from *some determinate Point* only. Nor indeed is it possible that he should do so. For if *this Picture*, ex. gr. upon such a *Table*, be an

an exact Representation of the *Life*; the *Life* it self, and the *Eye* which draws it, being in this or that *Position*; 'tis impossible that that same Picture, should be an exact Representation of the same *Life*, as it appears to that *Eye*, which is now shifted into a new Place or Position.

In a Word; together with the Reason of the thing, we may add this also; that one Point of Sight only, is to be found in the *Performances of the greatest Masters*, when 'tis a simple Design, and the Work consists but of one Piece.

III. *How to avoid Confusion, in setting Plans or Elevations, in Perspective.*

If when the *Plan* of any Figure is drawn in Perspective, it happens (thro' the *too great Obliquity* of the *Visual Lines*) that the Parts of it are crowd'd too close together, and by that Means become *confused* and indistinct; this may be easily remedied, by making choice of a new *Ground Line*, farther distant from the *Horizontal Line*, and so drawing a fresh Perspective Plan; which if not yet distinct enough, the Space between the Horizon and the Ground Line may be still enlarg'd, and so a new Plan drawn as before.

So

So likewise in *Elevations*, when by Reason of *too near an Approach to the Point of Sight*, the Projectures of the several Altitudes, cannot well be distinguish'd and design'd ; the Remedy for this Inconvenience, is by setting the Elevation at some due Distance, *farther from the Point of Sight* ; in which Case, the Parts which before were confus'd, by Reason of the too great Obliquity, will now become more obvious and distinct.

IV. *How deficient Figures, may be made to appear compleat, or any Figures may be made to appear of other Dimensions, than they really are ; by the Help of Perspective.*

'Tis suppos'd here, that some certain determinate Point is fix'd, from whence 'tis requir'd, that the Figures should appear compleat.

And the Perspective Work being done with Respect to that one Point ; it's impossible it should appear perfect, when view'd from any other Point but that alone.

Suppose a *Room* were of such a Figure, as wanted only one Angle or *Corner*, of a *true Square*, and this Defect were to be remedied by Perspective ; and the Room which is now a Trapezium, were to be made

made appear from a certain Point as it would, if it had been really Square.

If the *deficient Triangle*, be painted on the *Wall*, adjoyning thereto, as it ought to be by the Rules of Perspective, for the Eye in the Point assigned; I say then, that if the Place be viewed from that Point, it will appear, as if it had been a true Square.

If any Space were to be made appear *Longer* or *Broader*, than it really is, according to any Measure or Proportion assigned; this will be done by drawing according to the Rules of *vertical* Perspective, on the *Wall* or *Wainscoat*, at the farthest End, that Augmentation of the Area which is required.

As for Example, if an Area or Ground-floor were 40 Foot long and 10 broad; and it were to be made appear (keeping the same Width) as an Area of 60 Foot long: I say, that if an Area of 20 Foot, be painted in Perspective on the Wall or Wainscoat (as a *Table*) according to the *proper Distance*, from whence it is to be view'd; that then to an Eye fix'd in that Point, it will appear as if the Space it self were in reality so much longer.

The Reason of which is most obvious; for since by the Supposition, the *desired Increase of the Area*, is drawn in Perspective, according to the due Height and Distance

stance of the Eye, on a Table erected at the farthest End of the Space propos'd; that Piece of Perspective will undoubtedly appear to the Eye seated in its proper Point, just as the desired Prolongation of the Area it self would have done, if it had been true and real. And consequently, the whole together, *viz.* the real Area of 40 Foot, and the Perspective of the 20, upon the Wall; will appear, as an Area, of the real Dimensions of 60 would have done.

If the seeming Increase of Dimensions were to be *upwards*, instead of *long* or *broadways*; as if for Example, a Room or Entry, were to be made appear *higher* than it really is: In order to this Effect, we are to consider the *Roof* or *Cieling*, as a Table, and thereon to draw by the Rules of *Horizontal Perspective*, the Representation of what we would have, according to the intended Place and Seat of the Eye. Nor can there be any manner of Difficulty in any Practices of this kind, when the Agreement between *Horizontal* and *Vertical Perspective*, which we have demonstrated at *Prop. XXXVII.* is well consider'd.

By such like Artifices likewise (tho' not quite so easily, because it is more difficult to draw Pieces of Perspective accurately, upon *Inclin'd* Tables, than upon *Upright* Ones)

Ones) may *Inclining* or *Reclining Walls*, be made to appear Erect; and that painting on the said Walls, the true Scenographick Appearances, of those Parts of the Areas, of the Roof or Floor, which are deficient or redundant, from, or above, what would be taken in, by a true Perpendicular Position. For when a Wall *Inclines*, the under Pavement or Floor, is greater than the Ciëling; when it *Reclines*, 'tis less; when *Erect*, both are equal.

V. Of Lights and Shaddows.

Besides the rigorous *Mathematical* Part of Perspective, which shews upon demonstrative Principles, after what Manner the *Outlines* of Bodies are to be represented, or drawn upon a Table, for any given Height or Distance of the Eye; there is another, relating to the apt *Distinction of Lights and Shaddows*: Which depending much upon *Nature and Observation*, one may call (not improperly) the *Physical* Part of Perspective. 'Tis the Perfection of this Skill, that mainly surprizes in all Performances of this Kind. A just Mixture of Lights and Shaddows, without *accurate designing*, would not indeed please a Judge in these Matters; but the best *Design* in the World, with unnatural Lights and Shaddows, would

would scarce please any Body. It's less easie for a Fault (that is any thing gross) to lie concealed in *this Part*, than it is in the *other*; which is concern'd only in the drawing the Projection of Lines. For a Fault here, is a Fault against *sensible Nature*, which every one that observes, is a Judge of; but there, it is against *Mathematical Demonstration*, which few are conversant with.

How shocking would it be, to see in a Picture, a very, deep and strong Shaddow, together with a dark and cloudy Skie: Or the Lights let in, on the same Side, that the Shaddows fall of; when every Body knows, that the Light must necessarily come on the contrary Side!

These are Blunders which would easily be corrected by those, that might not be able to tell, *whether such and such Lines were rightly directed to their proper Point in the Picture, or no; or whether such a Column, or Tree, were aptly dispos'd, and of its just Height and Form.*

However, it may be useful to observe in these Cases, that besides the *Part or Quarter*, from whence the Light comes; the *Quality and Kind*, the *Altitude and Distance* of the Lights, and the *Manner* of the Illumination, is to be regarded.

It's one thing, to represent Bodies as enlightned by *Torches or Candles*, and another, as by the *Sun*.

Again, Things that are in the *open Air*, are not enlightned after the same Manner, as those that receive the Light, only thro' a *Hole or Window*.

Nor is an Object enlightned by *several Luminaries*, after the same manner, that it is by *one alone*. Nor when it is enlightned by a *Reflex'd or Reverberated Light*, is it to appear with that lively Brightness; as when it is expos'd to *open and direct* Radiations.

All these are to be consider'd, before a Man attempts to express the Effects of Illumination in a Picture. It's certain, for Example; that the *nearer* the Lucid Body is, by so much the more *Divergency* have the Rays of Light; and the *farther*, by so much the more do they approach, to a sensible *Parallelisme*. And therefore Illuminations by *Lamps*, and such like *very near* Luminaries, are to be express'd *Scenographically*; in such a Manner, that the *Shades* should be more plentiful, than the *Lights*. Whereas those which are caus'd by the *direct Rays of the Sun*, are to be design'd *Orthographically*, so as that the *Lights and Shades* should be equally distributed about. In like Manner, should the *Positions, Altitudes*

itudes and Distances of Luminaries, be consider'd, in Order to give Shaddows, their due Form, Proportions and Dimensions. Not forgetting the Effects of several Lights conjunctly illuminating a Body, whereby the Shaddow becomes fainter and more dilute, than when it is projected by one single Light alone; except where the Shaddows happen to intersect, one another; for there, viz. at the common Section, the Shade is always intense and deep.

Shaddows are of no less Use, in all the *Arts of Imagery and Representation*; than they are to the Purposes of *Astronomy and Geography*; where they serve to evince some of the most important Conclusions in both Sciences.

'Tis by *these* that we are sometimes led into Delusions, that are infinitely pleasant and agreeable to us. We mistake a little *Paint*, for *Life and Reality*; think a *flat Superficies*, sometimes to be a raised solid Body, and at other times to be hollow and depressed.

And so very fine and artful, are some Impostures of this Kind, that 'tis almost impossible for the best Judges to find them out; *pure Judgment* without particular *Acquaintance and Experience*, being not sufficient to correct the Errors, we run into upon such Appearances.

It won't be amiss to take Notice here, that *there are some Cases, wherein, without particular Consideration and Regard, had to the falling, both of the Lights and Shaddows; we are forc'd to remain in absolute Suspence about the true Form and Figure of a Body, whether it be Concave or Convex; really hollow and sunk in, or else elevated and Protuberant*: And this upon the Score of a real Ambiguity that there is in the Appearance; since the Body, which is thus represented, may be either Concave or Convex; and it is to be determin'd only from the Lights and Shaddows, which of the two it is.

Thus for Example; suppose there is a *Round* drawn, and *shaded* on one Side.

I am sure by the *Shade*, that it cannot possibly be a *Flat*, or a meer Circular Area, which is thus represented, but a *Solid*; but then whether it be *Hollow* or *Gibbous*, I cannot yet tell, without farther Consideration. But observing how the *Shaddow* is drawn, and at the same time knowing on which Side the *Light* falls; I can easily determine the Matter.

For if the *Light* falls on the Right Side (for Example) and the Picture be shaded on *that* same Side; I know then, that it must necessarily represent a *Concave*: But if the *Shade* be on the *opposite* Side, to that on which the *Light* comes; I am sure it must
express

express a *Convex*. For thus the Appearances would be, if a Solid, and a Hollow *Hemisphere*, were to be expos'd to the Light: The Protuberance of the *former*, would make it enlightened, and the sinking in of the *latter*, would make it shaded; on the *same Side* that the Light falls.

I need say nothing here, how the Figures, of the Shaddows of Bodies, are to be determin'd *Mathematically*. For I have shewn already, at *Schol. II. Prop. XXXIV.* that this Practice, is only the *Inverse*, of the Ordinary Perspective, and may be perform'd by the Rules.

However, those that please, may make use of the common Method; by *drawing Lines from the Light, and from the Foot of the Perpendicular, let fall from the Sight, to the Ground Plane*; which is in Effect the very same Method still.

Thus at Fig. VII. Prop. VII. If the Rectangle APBN, were an *Opake Body*, whose Shaddow were to be determin'd; the Light being at K, and its Altitude KV. The Lines KA, KB, extended from the Light, thro' the Angles A, B, and produced to meet in D, E, the Lines VP, VN, drawn from V, thro' the Angles P, N, in the Ground Plane; determine the Shaddow PDNE: Which is also a Piece of ordinary Perspective, in which DP, EN,

are *Radial Lines* running up to the *Point of Sight V* ; as we have demonstrated at Prop. XXXIV.

VI. Of Pictures in Pictures.

Whenever any Picture is represented as drawn in another Picture ; the Representation ought to be, according to the View of the real Spectator, who sees the *first* Picture with that *second* Picture in it ; and not according to the View of any Person drawn, in the *first* Picture, who is imagin'd to be a Spectator of the *second*.

Thus for Example, suppose a Person were to paint *Apelles*, drawing the Picture of *Alexander the Great*. He ought not in his Piece, to express the Picture of *Alexander*, as *Alexander* appear'd to *Apelles*, or according to *Apelles's* View ; but according to his own proper Image or Idea.

And the Reason is plain. For that Image of *Alexander*, which is a Copy to *Apelles* ; is an *Original* or *Archetype*, to our Painter : And consequently ought to be express'd by him, according to his own Idea.

And from hence a Judgment may be made (by those that are skilful this way) of the Defects or Perfections of many pompous Pieces, wherein Representations of this Kind are made. And

And thus I have said, what I propose to say of these Matters here.

I had design'd in this Treatise likewise, a particular Account of *Military Perspective*; or that which is made use of in the designing of *Fortifications*: But *this Sort of Perspective*, depending upon quite different Principles, from what the *Common* does; ought to be explain'd and illustrated, with Examples, by it self; which perhaps it may be, in another Place.

I shall here, at the Close of what has hitherto been said, of *Direct Vision*, subjoin one Problem, relating to *Refracted* and *Reflected Vision*.

I have formerly shown in another Place, [*viz.* *INSTITUT. FLUX.* Prob. 15.] how the *Foci* may be determin'd, for all Sorts of Glasses, of what Figure soever they are; receiving, either *parallel*, *Diverging* or *Converging* Rayes; and that by the Help of *one General Equation*, to be interpreted according to the particular Nature of each Curve: Which Method, I have since improv'd, in more Respects than *one*.

But what I intend here, is of a quite different Nature, *viz.* a *Geometrical Construction*; or an easie and accurate Method, of *tracing the Progress of a Ray by Scale and Compass*; and which I think to be different, from

from what I have seen, relating to this Matter.

Let MN (Fig. 38.) be some Refracting Circular Surface, denser than the ambient Medium; G the radiating Point; GAH the Axis produc'd thro' the Centre at Liberty; GNDP, the Incident Ray, produc'd at Liberty; AN drawn from the Centre A, to the Point of Incidence N.

Upon N, with the same Radius NA, strike an Arch AC; and from A, let fall AD, perpendicular to the Incident Ray ND, which produce till it cuts the Circle again in C; and draw NC.

The Quantities m , n , denoting the Proportion of the Sines of the Angle of Incidence and the Refracted Angle; make $NP : NC :: m - n : n$, and join PC. Let NQ Bisect the Angle PNC, and cut PC in Q; from whence let fall QB, perpendicular to DC.

Taking the Length of BC, in the Compasses, strike an Arch therewith upon the Centre of the Refracting Surface A.

Lastly, upon AN, as a Diameter, describe a Semi-Circle, cutting the foremention'd Arch in F.

I say, that laying a Ruler from N to F; it shall cut the Axis, in the Point H, which is the Focus sought: Or which is the same thing, the Line NF produc'd, is the true Refracted Ray. The

The *Demonstration* of which *Construction*, is as follows.

Because AD is *perpendicular* to GN in D, therefore $CND = AND =$ the Angle of Incidence; and so CD is the *Sine* thereof.

Farther, because $PN : NC :: m-n : n$, and the Angle PNC is bisected; therefore also $PQ : QC :: m-n : n$.

But since QB is perpendicular to AC, from thence NP and BQ are parallel;

Therefore $PQ : QC :: DB : BC$,

Therefore $DB : BC :: m-n : n$,

Therefore $DC : BC :: m : n$,

Therefore CB is = the *Sine* of the *Refracted Angle*.

Farther; because the Point F is determined by the Intersection of the Circle AFN, with the Circle whose Radius is = BC; therefore it's evident, not only that $AF = BC$, but also that the Angle AFN is a *Right* one, or AF perpendicular to FN.

Therefore NF is the true *Refracted Ray*, and H the Focus. *Q. E. D.*

Those that have a Mind to it, may investigate the Focal Distance, after the following or some such like Manner; having the Angle of Incidence, the Radius

dus of the Surface ; the Distance of the Radiating Point in the *Axis* from the Vertex of the Surface, and the Ratio of Refraction, all given.

Imagine (*it being omitted in the Figure*). a Perpendicular, let fall from the Point of Incidence N, to the Axis in R.

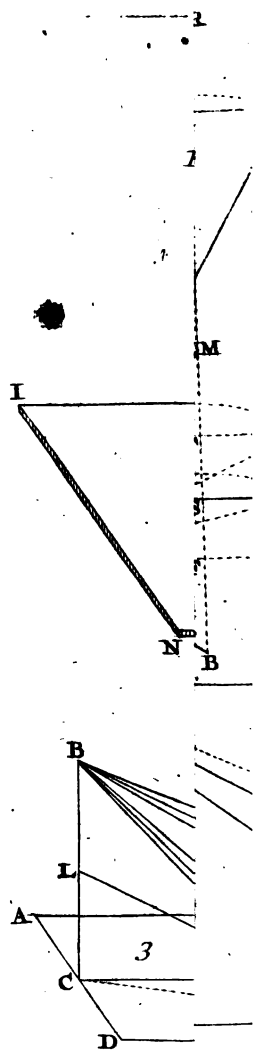
In the Right Angled \triangle^1 AND, there's AN and AND, given; whence AD and ND are given. Also MG being given,

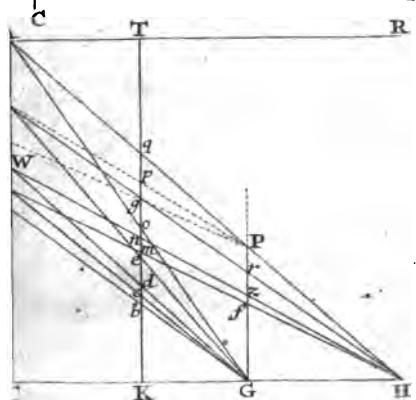
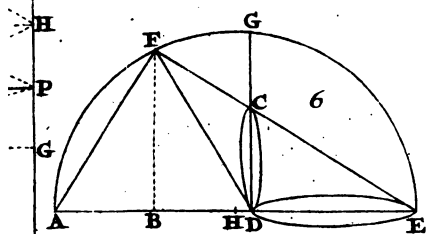
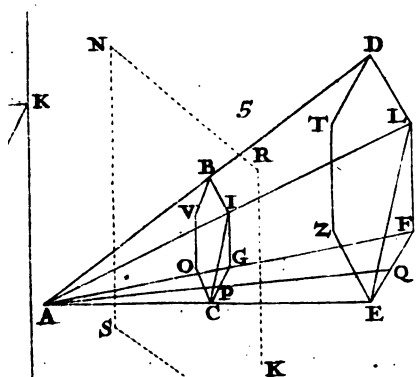
\therefore AG, and GD ($= \sqrt{AG^2 - AD^2}$) are given. \therefore also GN ($= GD - ND = \sqrt{AG^2 - AD^2} - \sqrt{AN^2 - AD^2}$) is

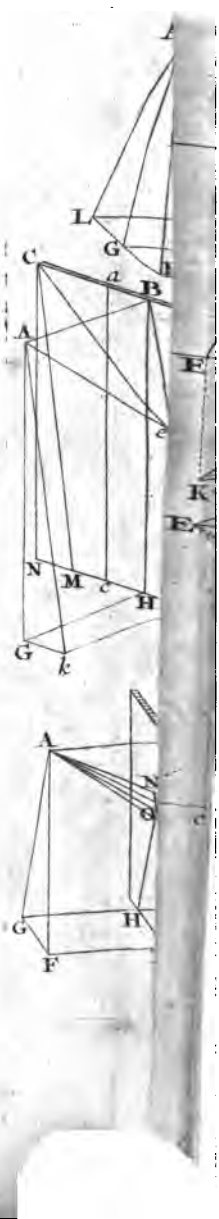
given. By Similar \triangle^1 GA : AD :: GN : RN, which is therefore given. Again, by Similar \triangle^1 GA : GD :: GN : GR, which is therefore given. Therefore, GA—GR=AR, is given also.

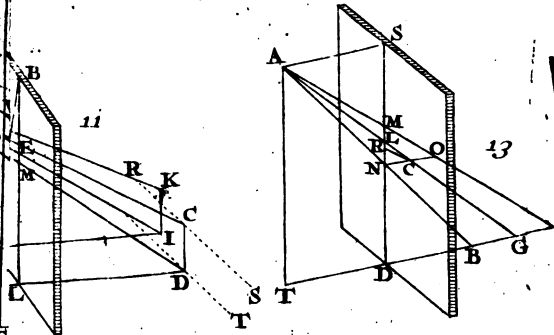
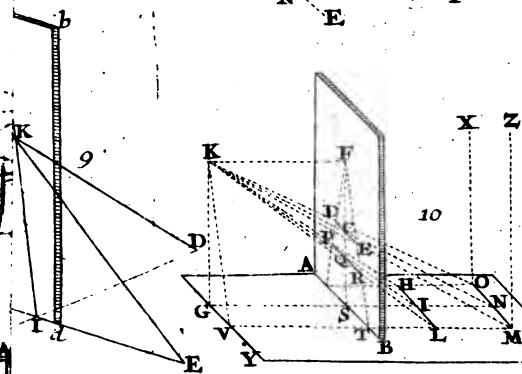
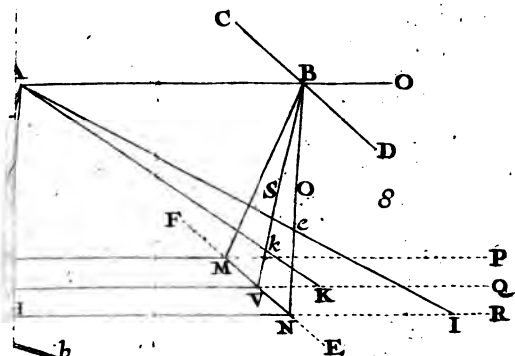
Lastly, The \triangle^1 RNH, AFH are Similar, therefore NR : RH :: AF : HF, that is, NR : RA+AH :: AF : $\sqrt{AH^2 - AF^2}$, whence $NR^2 \times AH^2 - NR^2 \times AF^2 = AF^2 \times RA^2 + AF^2 \times 2RA \times AH + AF^2 \times AH^2$, which gives but an *Affected Quadratick Equation*, for the finding of AH the Distance of the Focus, from the Centre of the Refracting Surface. Q. E. I.

Note 1. That this Construction and Investigation, suppose nothing, of the Rays falling near the Axis of the Refracting Surface,





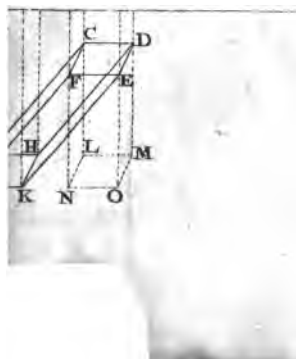
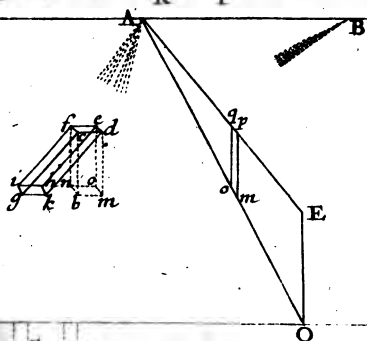
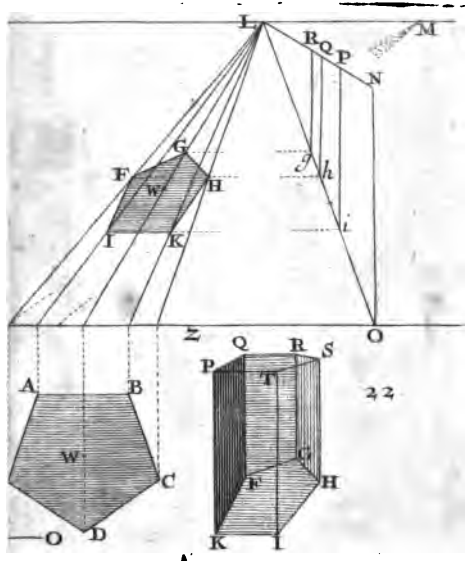




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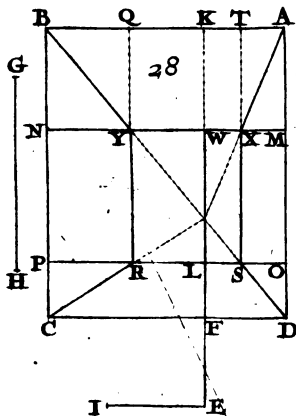
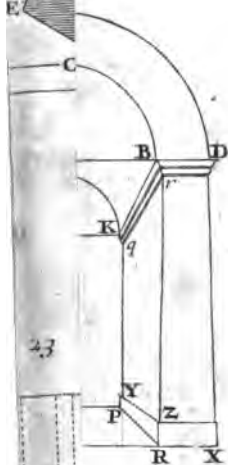
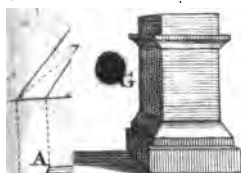
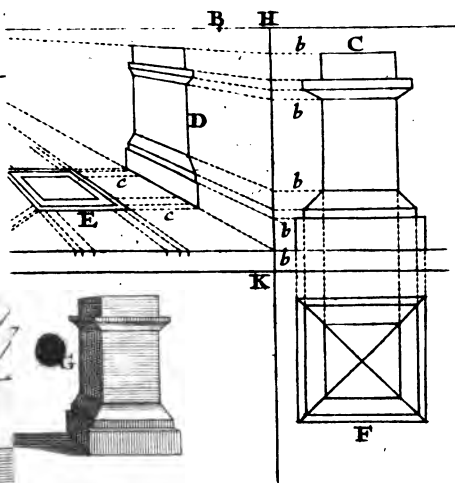
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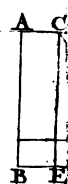
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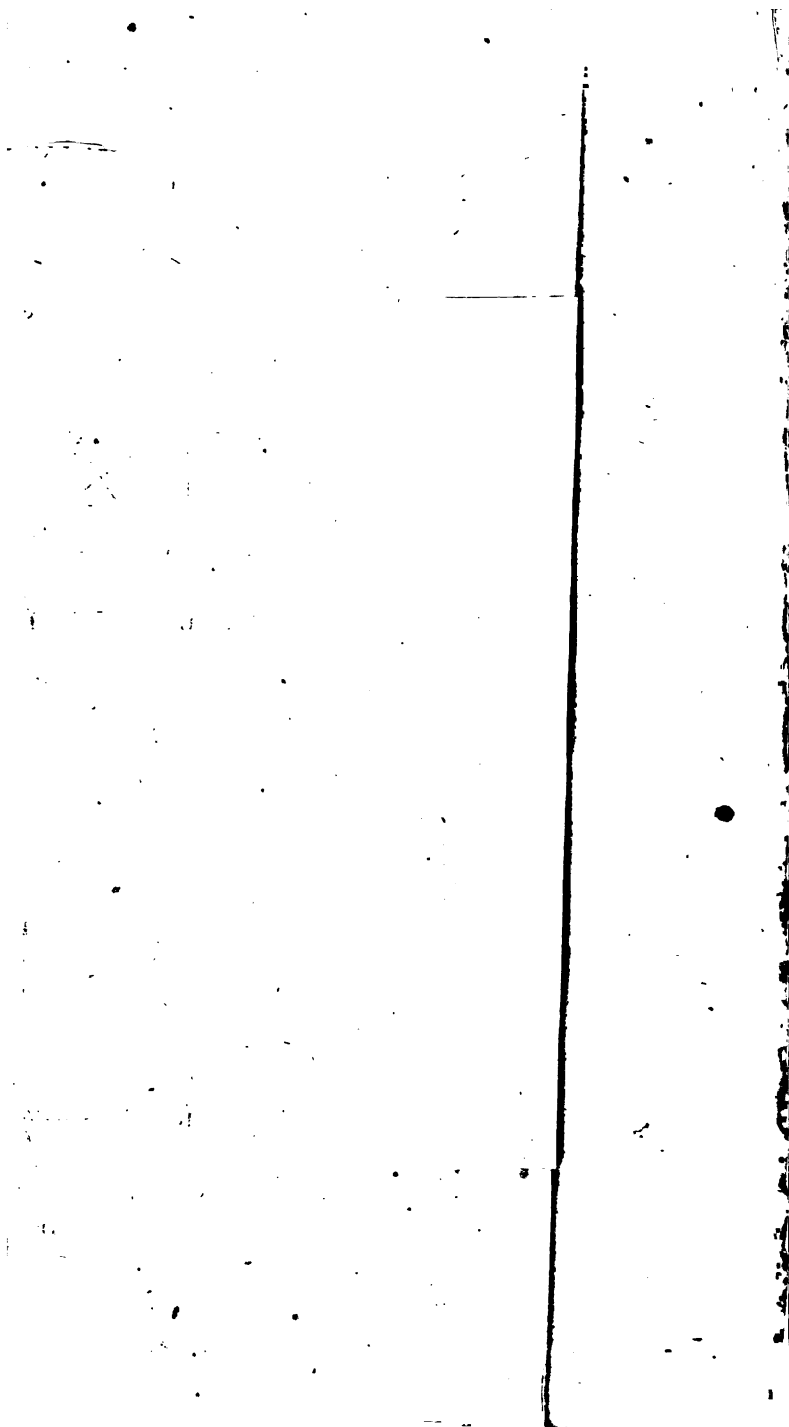
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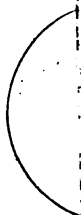
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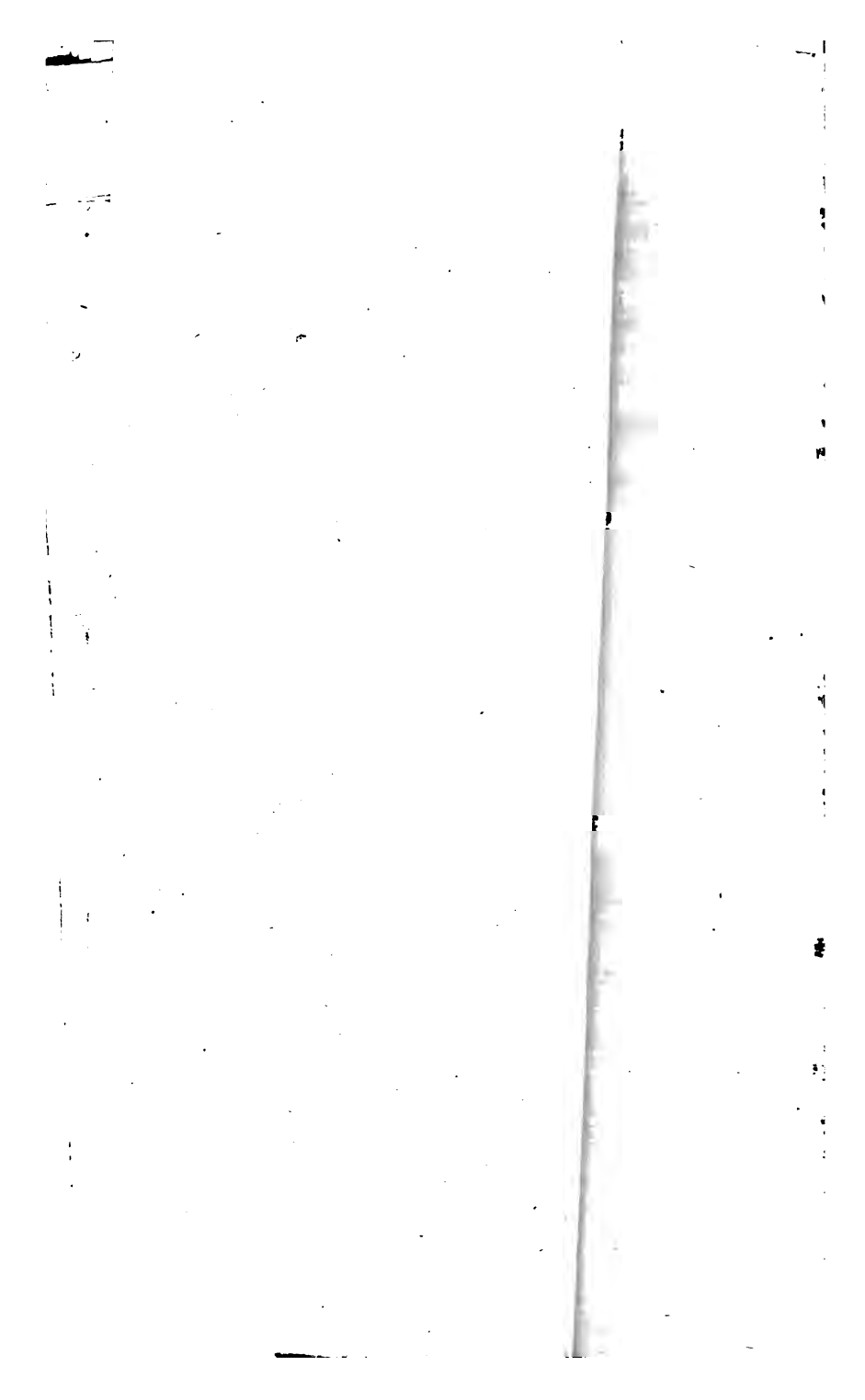


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face, but proceed all one, whether nearer or further off.

2. The Refracted Ray becomes either *parallel* (to the Axis) or *Converging*, or *Diverging*, that is, the Focus, is at an *Infinite*, *Finite*, or *more than Infinite* Distance, according as the Angle EAF, is *equal*, *more*, or *less*, than a Right one.

3. If the Curve MN were any other besides a Circle; its Property being given; by the *Methods of Tangents*, we can draw a Line, as AN *perpendicular* to the Curve, at the given Point of Incidence N; and then proceed as before. So that let the *Curvature*, be what it will, the Concourses of the Rayes as Refracted thereby, may be Practically and yet Geometrically trac'd out.

4. This Construction may easily be apply'd to any Sort of *Reflecting* Surface, as well as to *Refracting* ones; the Quantities *m*, *n*, which before were in the Ratio of the Refraction, being now put equal one to another.

F I N I S.